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MODELING AND DYNAMIC ANALYSIS OF THE PERFORMANCE OF DIFFUSER AUGMENTED WIND TURBINE

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Abstract: Increasing the diffuser area as well as the negative back pressure at the diffuser exit was found profitable in the experiments. The present paper aims to find a theoretical demonstration of DAWT by using theoretical analysis based on one-dimensional analysis, mathematical models, assumptions, estimations and maximization of power coefficients and augmentation ratios. With a simple momentum theory, developed along the lines of momentum theory for bare windturbines, it was shown that power augmentation is proportional to the mass flow increase generated at the nozzle of the DAWT. Modeling and analysis of DAWT system were estimated and improved for determining the power coefficient and augmentation ratio by changing area ratio, diffuser efficiency, velocity ratio, turbine factor and pressure recovery. Referred to rotor power coefficients values of $C_{p\ rotor} = 2.5$ might be achievable according to theory, It was apparent that an optimal exit-area-ratio would depend upon economic arguments however; it appears unlikely that the exit-area-ratio would exceed a value around 3.

Keyword: Diffuser flow; diffuser augmented wind turbine; theoretical demonstration; design and performance and one dimensional flow

النمذجة والتحليل الديناميكي لأداء Diffuser Augmented Wind Turbine

ملخص: مجردة. زيادة مساحة الناشر Diffuser Augmented Wind Turbine وكذلك الضغط السلبي العودة عند مخرج الناشر تم العثور على ربحية في التجارب. هذه الورقة تهدف الى ايجـــاد مظاهرة نظرية DAWT باستخدام التحليل النظري على أساس تحليل أحادية البعـــد ، والنمـــاذج الرياضية والافتراضات والتقديرات وتعظيم وزيادة نسب معامل الطاقة الكهربائية. وفيما نظريـــة الزخم بسيطة ، وضعت على غرار نظرية الزخم من أجل windturbines بدون ناشر ، تبين أن زيادة الطاقة بما يتناسب مع زيادة تدفق الشامل ولدت في فوهة للDAWT. وقدرت نمذجة وتحليل وتحسين نظام DAWT لتحديد معامل الطاقة وزيادة نسبة من خلال تغيير نسبة المساحة ، والكفاءة

الناشر ، ونسبة السرعة، وعامل ضغط التوربينات والانتعاش. وأشارت إلى قيم معامـــل الطاقـــة الدوار C=2.5 قد تكون قابلة للتحقيق وفقا للنظرية ، وكان من الواضح أن الأمثل للخـــروج مـــن منطقة إلى نسبة سيعتمد على الحجج الاقتصادية ومع ذلك ، بل يبدو من غير المرجح أن الخـــروج مـــــن منطقـــــــة إلـــــــى نـــــــسبة تتجـــــاوز قيمتهـــــا حـــــــوالي 3.

1. Introduction

Diffuser augmented turbines have recently gained new interest (Nash, 1998; Fly, 1999) [1],[2]. Based on experimental work in the seventies, where velocity augmentation ratio's of two even more were found (Oman, 1975; Foreman, 1979), sometimes enormous performances of DAWT's are quoted.

The first assessments on "ducted windmills" were performed by Lilley and Rainbird in 1956 [3]. In the seventies a significant amount of wind tunnel experiments were carried out by Foreman et al [4] from Grumman Aerospace Dept. USA, and by Kogan, Seginer and Igra [5] from the BenGurion University. Conclusions from these early studies varied significantly. The Lilley and Rainbird study concluded that no performance improvements were possible, when performance is referred to the exit area, where Foreman et al [3] conclude that "…the DAWT produced 25 times the power of the same turbine operating as a conventional turbine" and Igra [5] even concluded that "…the shrouded wind turbine is superior to any similar horizontal axis wind turbine of the same diameter". Apart from experiments these researchers also developed basic theoretical models to analyze their experiments. These models however lacked a coherent and complete description of the major phenomena in the flow field of a DAWT.

Enclosing a wind turbine by a diffuser reduces the pressure behind the turbine, thus enabling the drawing of more air for flow through the rotor, due to entrance velocity increase. Maximum power rating for a given blade technology can be extended substantially by installing a wind rotor at the entrance of a diffuser. The optimal performance of a wind turbine system has been studied (Badawy & Abd Raboo 1995; Mohamed & Badawy 1997), [6],[7] whereof the optimal power coefficients can be realized by the use of suitable pitch and twist angles in addition to design parameters such as tip speed ratio with minimum losses ($C_p \cong 0.435$). Gilbert et al (1977) made an experimental program guided by analytical models of various aspects of DAWT flow, where the power coefficient is 2.67 for a local turbine disk coefficient of 0.257. Forman et al (1976) [8], carried out a general study on the performance of DAWT system, whereby a diffuser area ratio of 3 gives maximum augmentation. Forman & Gilbert (1979) and Gilbert & Forman (1978) [9] carried out further investigations of DAWT and experimental demonstration for increasing augmentation. Turbomachinery diffuser design technology and review of diffuser analysis was studied by Japikse (1984), Mayer & Kneeling (1992) and Johnston (1998) [10], [11] and [12]. Finally the unified integral method has been demonstrated to produce results which are equal to or better than the performance parameters in terms of accuracy. It is obvious from the previous work, that the optimum configuration regarding numbers, positions, pressure recovery and overall factors of diffuser, suitable area ratio and the diffuser efficiency in addition to velocity ratio must be considered to get the optimum performance of a DAWT system.

Mertens [13] published a PhD thesis in 2006 where he addresses diffuser like concentrator effects that building may have on urban wind turbines and several new small DAWT wind turbines have entered the market

De Vries [14] was one of the first to develop a consistent theory for DAWT's. distinguishes simple diffuser theory from shrouded turbine theory. In the simple diffuser theory he models the one dimensional flow through a diffuser, but anticipated on the fact that the exit pressure should be equal to ambient. He incorporated negative backpressure values as found in the earlier experiments by introducing an empirical exit pressure coefficient. In his shrouded turbine approach he makes an attempt to model the radial forces on the shroud in a momentum approach. What was not found by De Vries was the fact that the optimal pressure drop over the rotor equals the optimal pressure drop over a bare wind turbine.

In 1999 Hansen et al [15] showed by means of CFD computations "…that the Betz limit can be exceeded with the ratio corresponding to the relative increase in mass flow through the rotor". A simple momentum theory for DAWT's was also derived, at that time by the author [16] and [17] using a number of straight forward assumptions. It is assumed that there is no viscous wake mixing process behind the diffuser but the effect of negative backpressures is taken into account in the performance prediction. From this DAWT momentum theory it can be seen that the achievable power is comparable with the power of a normal HAWT (horizontal axis wind turbine) having a diameter equal to the exit diameter of the diffuser. But from this momentum model it can also be seen that larger performances are possible when a substantial low "back pressure level" can be achieved at the diffuser exit.

This paper presents the well known one-dimensional momentum analysis for axi-symrnetric flow of a conventional bare turbine. A similar approach is then undertaken for the DAWT concept; however, it is shown that a closed theory cannot be established like that for the conventional bare turbine a theoretical demonstration of DAWT performance has been realized. Modeling and analysis of DAWT system were estimated and improved for determining the power coefficient and augmentation ratio by changing area ratio, diffuser efficiency, velocity ratio, turbine factor and pressure recovery.

2. One-dimensional analysis of a conventional bare wind turbine

2.1. Approach and assumptions

The one-dimensional analysis of a conventional bare turbine is based around the assumption that the turbine is represented as an actuator disc. The disc is considered ideal, that is, it is frictionless and there is no rotational velocity component in the wake. The entire process is assumed to occur at a small Mach number and the air density is thus constant. The flow is also assumed to be steady, incompressible, and frictionless and there are no external forces acting on the fluid up- or down-stream of the turbine.

2.2. Modeling and simulation

The depiction in Figure 1 shows that the actuator disc acts as a drag device slowing the wind speed down in a continuous manner from V_∞ far upstream of the rotor, to V_r at the rotor and eventually V_5 far downstream in the wake. The retardation of flow gives rise to a divergence in the streamtube passing through the periphery of the disc. There is an associated pressure rise on the upstream side of the rotor to a value P_2. Downstream of the rotor the opposite occurs with a gradual pressure recovery continuing until the level returns to atmospheric, P_∞ in the far wake. Across the actuator disc there is a discontinuous pressure drop from P_2 to P_3.

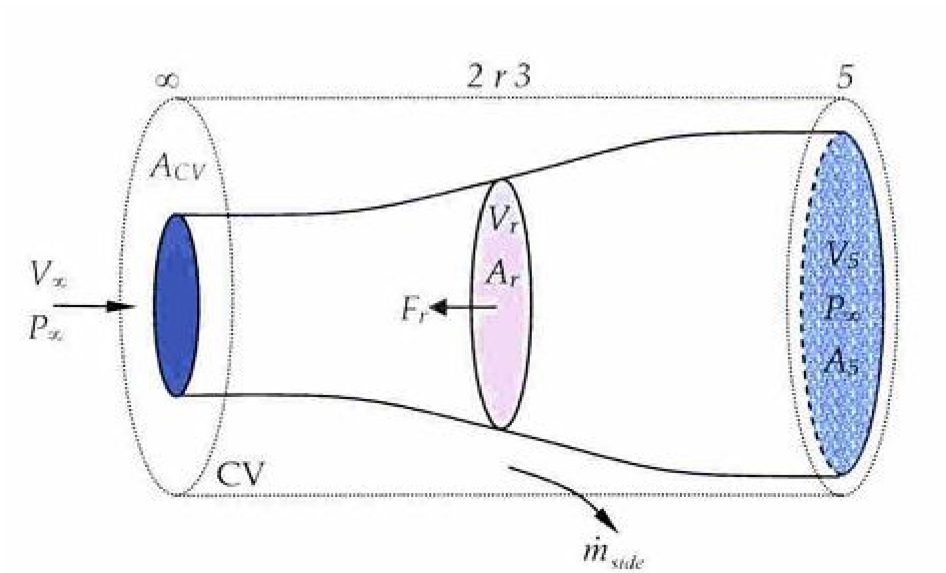

Figure 1: Streamtube passing through a conventional bare turbine

These changes in pressure and velocity upstream and downstream of the actuator disc can be described by the Bernoulli equation under the assumptions used in the one- dimensional analysis. The equation for the streamtube beginning far upstream and ending at station *2* just in front of the rotor is given by

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p_2 + \frac{1}{2}\rho V_2^2 \qquad (1)$$

Likewise, the equation describing the flow from just behind the rotor at station 3 to far downstream in the wake is given by

$$p_3 + \frac{1}{2}\rho V_3^2 = p_\infty + \frac{1}{2}\rho V_5^2 \qquad (2)$$

Across the rotor, the thrust, *Fr*, that is the force in the streamwise direction resulting from the pressure drop across the turbine, can be writhen as

$$F_r = (p_2 - p_3)A_r \qquad (3)$$

2.2.1. Energy balance

It is now possible to determine the power output of the actuator or in other words, the rate of energy loss across if by combining equations (1), (2) & (3) and multiplying by V_r. This represents the energy balance between stations ∞

and 5. It should be noted that $Q_2 = Q_r = Q_\infty$ and therefore as volume flux across sections of the bounding streamtube, The power output of the actuator is therefore given by $Q = A_1 V_1$, is constant due to conservation of mass $V_2 = V_r = V_3$. The power output of the actuator is therefore given by

$$Q\left(\frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V_5^2\right) = V_2 F_r = P_{air} \tag{4}$$

The energy absorbed by the turbine can then be normalized and written in terms of $C_{p\,air}$ by dividing equation 4 by $\frac{1}{2}\rho V_\infty^3 A_2$yielding

$$Cp_{air} = \frac{P_{air}}{\frac{1}{2}\rho V_\infty^3 A_2} = \frac{V_2}{V_\infty}\left(1 - \frac{V_5^2}{V_\infty^2}\right) \tag{5}$$

2.2.2. Axial momentum balance

Considering the cylindrical control volume shown in Figure 1, the axial momentum balance can be writhen in integral form as

$$\frac{\partial}{\partial t}\iiint_{CV} \rho U d(vol) + \iint_{CS} U\rho V.\, dA = F_{ext} + F_{pres} \tag{6}$$

where dA is a vector pointing in the normal direction of an infinitesimal part of the control surface with a length equal to the area of this element. F_{pres}, is the axial component of the pressure forces acting on the control volume and $d(vol)$ denotes an incremental part of the control volume.

Under the assumption of steady flow, the first term of (6) becomes zero. As the pressure on the upstream and downstream ends of the control volume are equal to the same atmospheric level and act over equal areas, F_{pres} is also zero. There is no axial contribution from the pressure on the lateral boundary of the control volume. Using the simplified assumptions of an ideal rotor, equation (6) then yields

$$\rho V_5^2 A_5 + \rho V_\infty^2 (A_{CV} - A_5) + \dot{m}_{side} V_\infty - \rho V_\infty^2 A_{CV} = -F_r \tag{7}$$

The mass-flow rate through the lateral boundary can then be found from the conservation of mass to be

$$\dot{m}_{side} + \rho V_5 A_5 + \rho V_\infty (A_{CV} - A_5) = \rho V_\infty A_{CV} \tag{8}$$

Yielding

$$\dot{m}_{side} = \rho A_5 (V_\infty - V_5) \tag{9}$$

Equations (7) & (9) can be combined to define conventional bare turbine to be the momentum balance for the

$$F_r = \rho Q (V_\infty - V_5) \tag{10}$$

or in coefficient from by dividing by $\frac{1}{2}\rho V_\infty^2 A_2$:

$$Ct_\infty = 2\frac{V_2}{V_\infty}\left(1 - \frac{V_5}{V_\infty}\right) \tag{11}$$

Comparing equation (10) with the energy balance (4) yields, the well known result

$$V_2 = \frac{1}{2}(V_\infty - V_5) \tag{12}$$

that is, the velocity at the rotor is equal to the mean of the wind speed far upstream, V_∞ and the final velocity in the wake V_5.

By rearranging (12) in terms of the wake velocity, V_5, and substituting the result into (11). the thrust coefficient can be defined in terms of the velocity speed-up ratio, ε, that the ratio of velocity across the blade-plane, V_2, to the velocity far upstream, V_∞;

$$Ct_\infty = 2\left(\frac{V_2}{V_\infty}\right)\left(1 - \frac{2V_2 - V_\infty}{V_\infty}\right) \tag{13}$$

$$= 4\varepsilon(1 - \varepsilon)$$

The same substitution this time into the same into equation (5) yields the power coefficient defined in terms

$$Cp_{air} = \frac{V_2}{V_\infty}\left(1 - \frac{(2V_2 - V_\infty)^2}{V_\infty}\right)$$

$$= 4\varepsilon^2(1 - \varepsilon) \tag{14}$$

When the velocity at the actuator disc, V_2 is defined as is traditionally done for the conventional bare turbine case in terms of the axial induction factor, *a*:

$$V_2 = V_\infty(1-a) \tag{15}$$

the classic solutions for the thrust coefficient

$$Ct_\infty = 4a(1-a) \tag{16}$$

along with that for the power coefficient

$$Cp_{air} = 4a(1-a)^2 \tag{17}$$

Are obtained

From this point, it is now possible to determine the theoretical maximum power output of a conventional bare turbine by differentiating Cp_{air} with respect to *a*. The result yields

$$\frac{dCp_{air}}{dx} = 4(1-a)(1-3a) \tag{18}$$

from which it can be seen that a maximum is obtained for $a = \frac{1}{3}$. Substitution of this into (17) yields the theoretical maximum power output of a conventional bare turbine, $Cp_{air\,max} = \frac{16}{27}$, known as the Betz limit. Similarly, substitution into (16) shows this maximum to be obtained at a thrust coefficient of $Ct_\infty = \frac{8}{9}$.

For later comparison, the pressure immediately downstream of the rotor at station 3 will be formulated. Using Bernoulli's equation for the flow upstream of the rotor, the pressure difference between station 2 and station ∞ relative to the dynamic pressure in the upstream flow for a conventional bare turbine operating at the Betz limit is

$$\frac{P_2 - P_\infty}{\frac{1}{2}\rho V_\infty^2} = \left(1 - \frac{V_2^2}{V_\infty^2}\right) = \frac{5}{9} \tag{19}$$

Adding to this the pressure drop across the rotor, the pressure coefficient immediately downstream of the rotor is found to be

$$Cp_3 = \frac{P_2 - P_\infty}{\frac{1}{2}\rho V_\infty^2} + \frac{P_3 - P_2}{\frac{1}{2}\rho V_\infty^2}$$

$$= \left(1 - \frac{V_5^2}{V_\infty^2}\right) + Ct_\infty$$

(20)

$$= -\frac{1}{3}$$

3. One-dimensional analysis of a DAWT

A similar approach can be taken for the analysis of a DAWT using the same assumptions of an ideal actuator disc with a non-rotational wake with steady, incompressible and frictionless flow at small Mach numbers. The difference, however, is that external forces act on the fluid upstream and downstream of the turbine as shown in Figure 2. It is also assumed that the flow remains attached throughout the diffuser separating only from the diffuser's hailing edge.

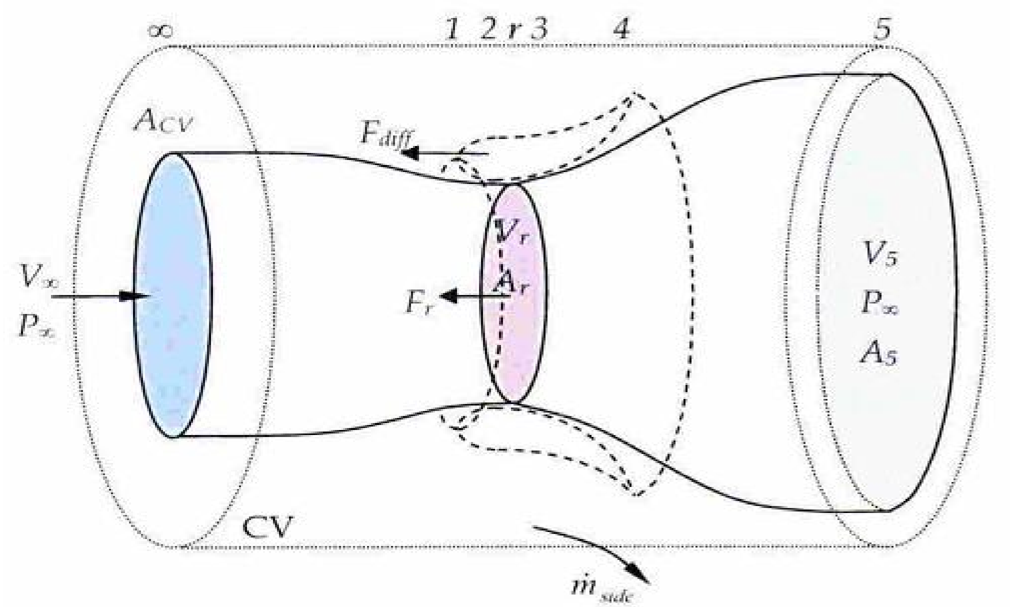

Figure 2: Streamtube passing through a ducted turbine rotor

The flow upstream of the rotor can be described by the Bernoulli equation assuming that there are no inlet losses for a well designed DAWT.

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p_2 + \frac{1}{2}\rho V_2^2 \qquad (21)$$

Similarly, the Bernoulli equation holds for the flow downstream of the diffuser exit

$$p_4 + \frac{1}{2}\rho V_4^2 = p_\infty + \frac{1}{2}\rho V_5^2 \qquad (22)$$

The thrust on the rotor is again given by

$$F_r = (p_2 - p_3)A_r \qquad (23)$$

3.1 Energy balance

The energy balance can now be formulated for the DAWT. Unlike the conventional bare turbine case, the energy balance for the DAWT must include the losses through the diffuser which are given by

$$\Delta H_{diff} = p_3 - p_4 + \frac{1}{2}\rho\,(V_3^2 - V_4^2) \qquad (24)$$

The energy balance for the ducted ease is therefore

$$Q\left(\frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V_5^2\right) = V_r F_r + Q\Delta H_{diff} \qquad (25)$$

A, more useful form of this can be obtained by relating the diffuser losses, ΔH_{diff} to the diffuser efficiency η_{diff} is defined as

$$\eta_{diff} = \frac{p_4 - p_3}{\frac{1}{2}\rho(V_3^2 - V_4^2)} \qquad (26)$$

The diffuser losses therefore become

$$\Delta H_{diff} = \left(1 - \eta_{diff}\right)\frac{1}{2}\rho(V_3^2 - V_4^2)\ or: \Delta H_{diff} = (p_4 - p_3)\left(\frac{1-\eta_{diff}}{\eta_{diff}}\right) \qquad (27)$$

Incorporating this into equation diffuser efficiency (25) yields the power output for the DAWT in terms of

$$P_{air} = V_r F_r = Q\frac{1}{2}\rho\left(V_\infty^2 - V_5^2 - (1-\eta_{diff})(V_3^2 - V_4^2)\right) \tag{28}$$

As per the conventional bare turbine case this can be described in a non-dimensional form by dividing $\frac{1}{2}\rho V_\infty^3 A_r$ where

$\lambda = \frac{A_5}{A_4}$ is the reciprocal of the diffuser exit-area-ratio and

$\varepsilon = \frac{V_2}{V_\infty}$ is the velocity speed-up ratio.

Thus, the power coefficient for the DAWT is given by

$$Cp_{air} = \varepsilon\left(1 - \frac{V_5^2}{V_\infty^2} - (1-\eta_{diff})(1-\lambda^2)\varepsilon^2\right) \tag{29}$$

Rather than having the power output defined in terms of the velocity in the far wake, Vs, it is preferable to use the diffuser exit pressure coefficient, Cp_4 where , Cp_4 is defined as

$$Cp_4 = \frac{p_4 - p_\infty}{\frac{1}{2}\rho V_\infty^2} \tag{30}$$

By using mass conservation between stations 3 and 4, it can be shown that

$$V_4 = \lambda\,\varepsilon V_\infty \tag{31}$$

Substituting (31) into the Bernoulli equation downstream of the diffuser exit (22), the velocity in the far wake, V, can be shown to be given by the relationship

$$\left(\frac{V_5}{V_\infty}\right)^2 = \lambda^2\varepsilon^2 + Cp_4 \tag{32}$$

When combined with equation (29), the final result becomes

$$Cp_{air} = (1 - Cp_4)\varepsilon + (\eta_{diff}(1-\lambda^2) - 1)\varepsilon^3 \tag{33}$$

To prove the pressure dependence on velocity speed-up and disc loading by applying the same method to the conventional bare turbine case with *ε= (1 - a)* and λ=1 in equation (33). This simply implies that the diffuser does not expand, but is a constant area duct.

$$Cp_{air} = 4a(1-a)^2 = (1-Cp_4)(1-a) + \left(\eta_{diff}(1-1^2)-1\right)(1-a)^3$$
(34)

which yields

$$Cp_4 = a(3a-2) \tag{35}$$

3.2 Axial momentum balance

It should be noted that the result for the bare turbine evolves from using the energy equation in conjunction with the momentum equation and the fact that there is no net contribution to axial momentum from pressure forces on the free-streamlines. If, however, the axial pressure forces on the bounding streamtube upstream of the turbine and these on the downstream part are considered separately, they can be shown to be non-zero; equal in magnitude and opposite in sign. Their magnitude is obtained by considering a small perturbation in pressure from the free-stream pressure acting on the control volumes upstream and downstream of the turbine respectively and using
$V_2 = \frac{1}{2}(V_\infty + V_5)$.

Thus for a bare turbine, f we first consider the momentum balance upstream of the rotor, then the upstream force, F_{us} is given by

$$F_{us} - (p_4 - p_\infty)A_2 = \rho Q(V_2 - V_\infty)$$

$$p_2 - p_\infty = \frac{1}{2}\rho(V_\infty^2 - V_2^2)$$
(36)

$$F_{us} = \rho Q(V_2 - V_\infty) + \frac{1}{2}\rho(V_\infty^2 - V_2^2)A_2$$

and in terms of the induction factor a:

$$F_{us} = \frac{1}{2}\rho V_\infty^2 A_2 a^2$$
(37)

Similarly, downstream of the turbine the momentum balance is

$$(p_3 - p_\infty)A_3 + F_{ds} = \rho Q(V_5 - V_3)$$

$p_3 - p_\infty = \frac{1}{2}\rho(V_5^2 - V_3^2)$

(38)

$F_{ds} = \rho Q(V_5 - V_3) - \frac{1}{2}\rho(V_5^2 - V_3^2)A_3$

and again in terms of the induction factor a:

$$F_{ds} = -\frac{1}{2}\rho V_\infty^2 A_3 a^2 \qquad (39)$$

These expressions can then be described in terms of different areas and/or velocities by the well-known relations between them.

For the DAWT, the magnitude and sign of the axial pressure forces on the bounding streamline are not necessarily equal and opposite. In fact, the situation can exist in which the axial component of the pressure on the upstream free-streamline equals zero (for $\varepsilon = 1$), while there is still post expansion that is $Cp_4 < 0$. This will lead to a net pressure force on the free-streamline between stations 4 and 5. The summation of the momentum balances between each station should be combined with the energy balance as performed on the conventional bare turbine in determining the theoretical maximum performance of a DAWT.

Again the magnitude of the axial pressure forces on the bounding streamtube is obtained by considering a small perturbation in pressure from the free-stream pressure acting on the control volumes.

For the upstream free-streamtube

$F_{us} + p_\infty A_\infty - p_2 A_2 - p_\infty(A_\infty - A_2) = \rho Q(V_2 - V_\infty)$

$F_{us} - (p_2 - p_\infty)A_2 = \rho Q(V_2 - V_\infty) = \rho V_\infty^2 A_2 \varepsilon(\varepsilon - 1)$

$p_2 - p_\infty = \frac{1}{2}\rho(V_\infty^2 - V_2^2) = \frac{1}{2}\rho V_\infty^2(1 - \varepsilon^2)$

(40)

$F_{us} = \frac{1}{2}\rho V_\infty^2 A_2(1 - \varepsilon)^2$

This is identical to equation (36) with $\varepsilon = (1 - a)$

Across the rotor, the force on the air is

$$F_r = -(p_2 - p_3)A_r \qquad (41)$$

The force from the diffuser on the streamtube passing through it is obtained from the integral of the axial component of pressure forces over the inner surface given by

$$F_{diff} = \iint_R Pdrd\theta \tag{42}$$

which can be shown to be

$$F_{diff} = \left(p + \frac{1}{2}\rho V^2\right)(A_4 - A_3) - \frac{1}{2}\rho Q^2 \left(\frac{A_4 - A_3}{A_4 A_3}\right) \tag{43}$$

for a diffuser with an efficiency of 100%. For realistic efficiency values, this formula has to be corrected.

For the post-expansion free-streamline

$$p_4 A_4 - p_\infty A_5 + p_\infty (A_5 - A_4) + F_{ds} = \rho Q(V_5 - V_4)$$

$$\frac{p_4 - p_\infty}{\frac{1}{2}\rho V_\infty^2} = Cp_4 = \frac{\frac{1}{2}\rho V_5^2 - \frac{1}{2}\rho V_4^2}{\frac{1}{2}\rho V_\infty^2}$$

$$Cp_4 = \left(\frac{V_5}{V_\infty}\right)^2 - \left(\frac{V_4}{V_\infty}\right)^2 = \left(\frac{V_5}{V_\infty}\right)^2 - \left(\frac{V_4}{V_2}\frac{V_2}{V_\infty}\right)^2 = \left(\frac{V_5}{V_\infty}\right)^2 - \lambda^2\varepsilon^2$$

(44)

$$\frac{V_5}{V_\infty} = [Cp_4 + \lambda^2\varepsilon^2]^{1/2}$$

And finally

$$F_{ds} = \frac{1}{2}\rho V_\infty^2 A_2 \left[2\varepsilon\{[\lambda^2\varepsilon^2 + Cp_4]^{1/2} - \lambda\varepsilon\} - \frac{Cp_4}{\lambda}\right] \tag{45}$$

The final monmentum balance is obtained by summation of all terms:

$$F_{us} + F_r + F_{diff} + F_{ds} = \rho\varepsilon V_\infty A_2 (V_5 - V_\infty) \tag{46}$$

Multiplication by V_2 and substituting $V_2 F_r = P_{air}$ from the energy equation (28), a relation between ε and Cp_4 is obtained. To cast this in a useful form, (43) must first be expressed including effects of diffuser efficiency.

The theory can be used to examine the relative influence of each parameter on DAWT performance, identify those parameters on which to focus during the development of DAWT designs.

4. Simulation results and discussion

The key parameters relating to DAWT performance are shown in equation (33) to be the exit pressure, Cp_4, the velocity speed-up, ε the diffuser efficiency, η, and the diffuser exit-area-ratio, the reciprocal of which is λ. It is also important to note that the power output is dependent on the rotor thrust, referred to as the free-stream disc loading coefficient, Ct_∞. The disc loading is relative to the local velocity at the blade plane, V_2. The local disc loading coefficient is therefore defined as

$$Ct_\infty = \frac{\Delta P_{2-3}}{\frac{1}{2}\rho V_2^2} \quad (47)$$

with the relationship between free-stream and local disc loading coefficients being

$$Ct_\infty = Ct_2 \varepsilon^2 \quad (48)$$

It has also been common for the DAWT performance to be augmentation relative to the Betz limit, that is described by the

$$r_{air} = \frac{Cp_{air}}{0.593} \quad (49)$$

To ascertain the influence of each DAWT performance parameter, it is possible to produce a range of augmentation curves by using the theoretical analysis above along with prescribed values for the exit pressure coefficient, diffuser efficiency, diffuser exit-area-ratio and disc loading coefficient, an example of which is shown in Figure 3. The values for the exit pressure coefficient and exit-area-ratio used in this example are those obtained by Grumrnan's DAWT in [4].

It can be seen in this figure that the augmentation is critically dependent upon the diffuser efficiency. It is also apparent that the diffuser efficiency impacts upon the optimal local disc loading coefficient with a lower disc loading coefficient occurring for increased diffuser efficiency.

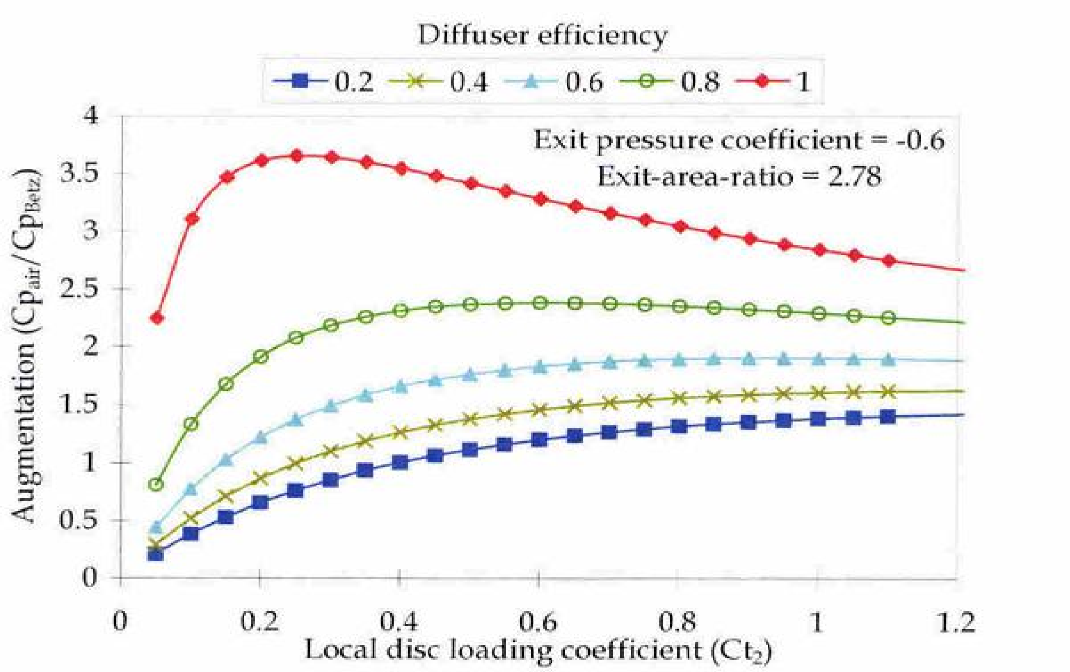

Figure 3: influence of diffuser efficiency on DAWT augmentation over a range of local disc loading coefficients.

It is therefore of interest to investigate the impact each parameter has on augmentation along with the relationships between them. The influence of diffuser efficiency touched on above is discussed first followed by that resulting from exit-area-ratio and exit pressure coefficient.

The diffuser efficiency can be seen in Figure 4 to have a marked impact on augmentation. It is apparent that this increase is due to the significant increase in velocity speed-up that occurs for diffuser efficiencies above about 80%. Although a positive linear trend between optimal local disc loading coefficient is shown with increased diffuser efficiency, the free-stream disc loading coefficient appears to be independent of diffuser efficiency. This suggests that the optimal operating conditions within the DAWT are unaltered by diffuser efficiency and that the increase in augmentation is a result of increased mass-flow through the blade-plane.

Increased exit-area-ratio also produces an increase in augmentation due to enhanced mass-flow through the DAWT. Figure 5 show that velocity speed-up increases as the exit-area-ratio increases whilst the free-stream disc loading coefficient remains constant.

The trend however shows that unlike the exponential style growth in augmentation evident with increased diffuser efficiency, the rate of increase in augmentation reduces for larger exit-area-ratios. This hints towards an optimal exit-area-ratio existing for DAWT's beyond which cost increases outweigh the increase in augmentation.

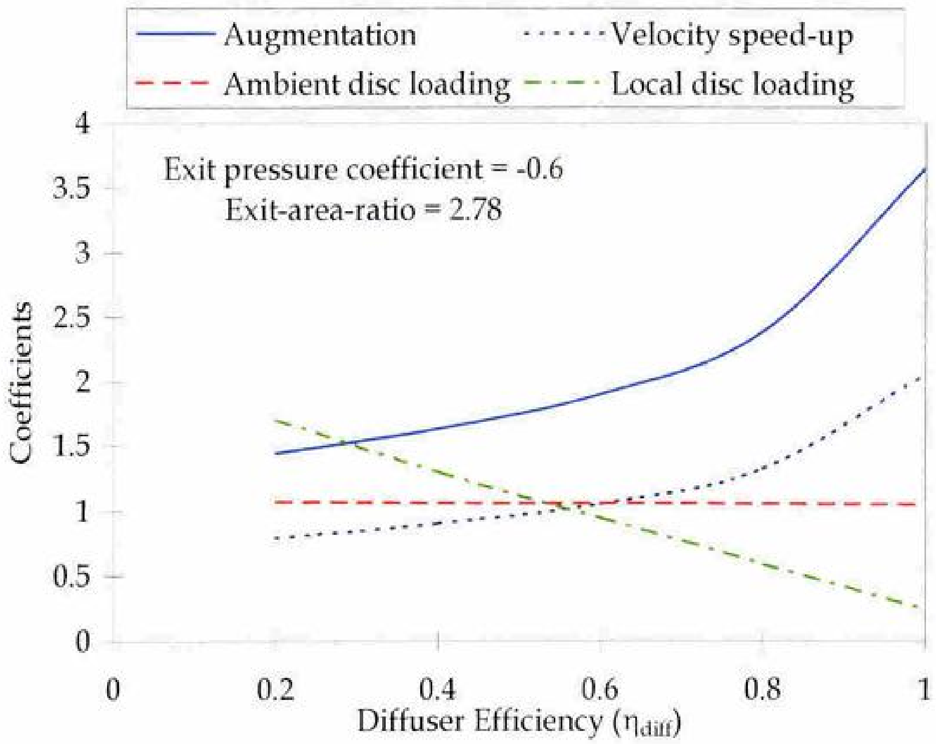

Figure 4 Influence of diffuser efficiency on augmentation, velocity, speed-up free-stream and local disc loading coefficients.

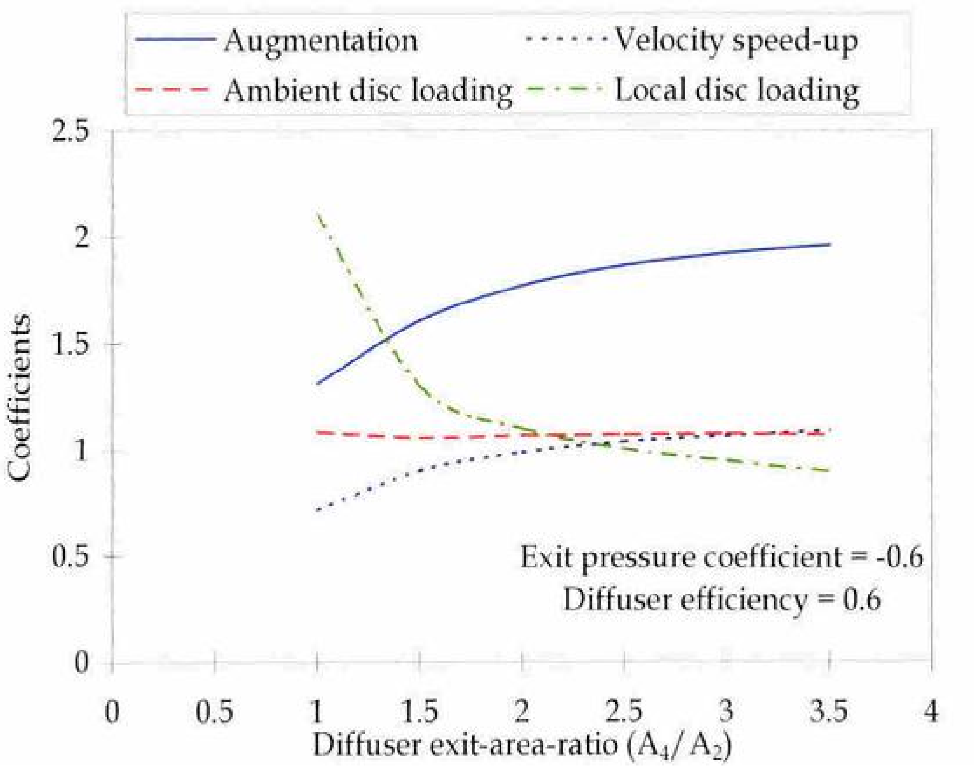

Figure 5: Influence of exit area-ratio on augmentation, velocity speed-up, free-stream and local disc loading coefficient

Finally, the influence of exit pressure can be seen to not only have linear relationships with augmentation and velocity speed-up but also with the free-stream disc loading coefficient (Figure 6). This relationship between exit pressure coefficient and free-stream disc loading is of significance as it suggests that if a DAWT is capable of creating an exit pressure coefficient more sub-atmospheric than that found in the near-wake of a conventional bare turbine. An increase in free-stream disc loading equates to an increase in pressure drop across the blade-plane and therefore an increase in the energy captured per unit of mass-flow passing through the DAWT.

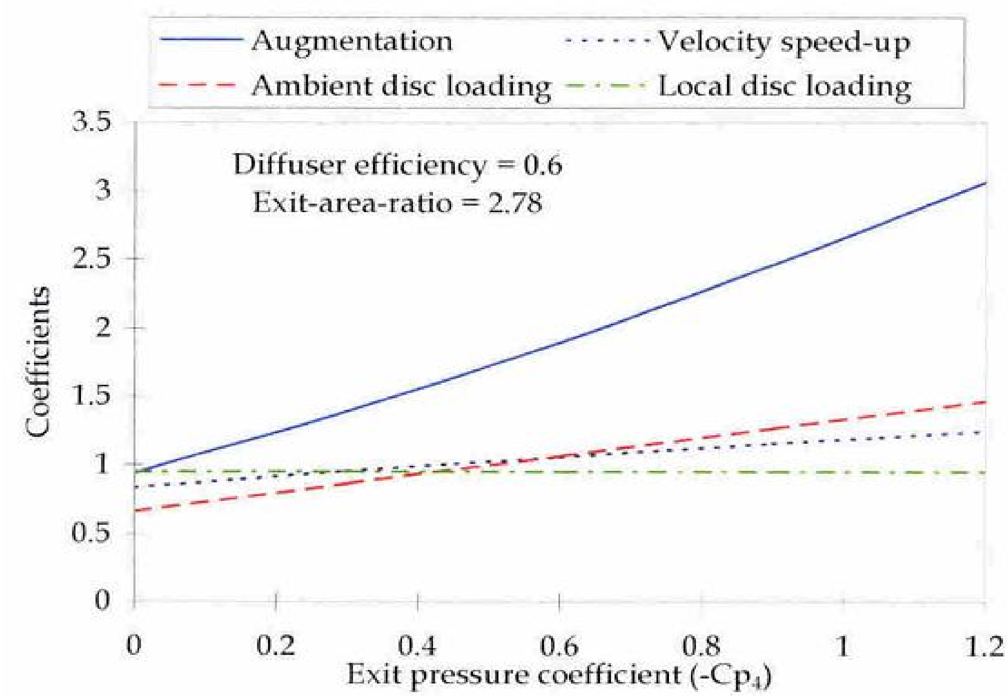

Figure 6: Influence of exit pressure coefficient on augmentation, velocity speed-up, free-stream and local disc loading coefficients.

Having demonstrated the relationships between the various parameters, it is of interest to examine the relative impact a change in each parameter has on augmentation. The impact was ascertained by varying each parameter individually whilst using constant reference values for the other parameters.

Figure 7 shows that, the two parameters having the most significant influence on augmentation are the diffuser efficiency and the exit pressure coefficient. In contrast, the exit-area-ratio shows only a minor influence on augmentation above a value of approximately 3. This concurs with the result obtained experimentally by Grumman [19] where the optimal exit-area-ratio was identified to exist around a value of 3. Figure 7 also shows that although an optimal disc loading exists, the impact on augmentation of DAWT operation either side of this disc loading is relatively small, particularly so for disc loadings above optimal.

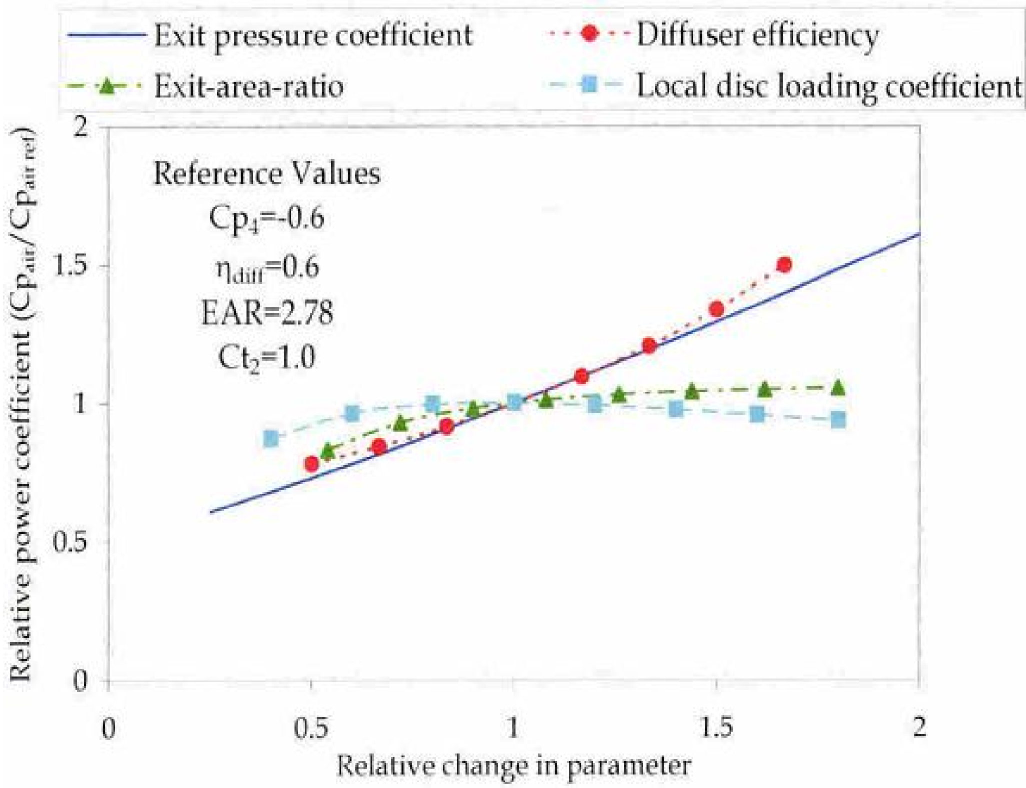

Figure 7: Effect of relative changes of the parameters governing DAWT performance [2].

5. NUMERICAL EXAMPLE

In the following it is assumed that a diffuser area ratio of β=2 is established and that the turbine inside the diffuser operates at optimal condition (a = 1/3). At first it is assumed that no back pressure is present behind the empty diffuser. According to (8) the velocity fond at the wind turbine then equals:

$$V_1 = \frac{4}{3}V_0 \tag{50}$$

And the pressure in font of the turbine

$$p_1 = p_0 + \frac{1}{2}\rho[1 - \beta^2(1 - \alpha)^2]V_0^2 = p_0 - \frac{7}{9}\frac{1}{2}V_0^2 \tag{51}$$

So instead of over pressure (at a standard wind turbine) we now have already under pressure in front of the turbine inside the diffuser.
Since there is a pressure drop over the turbine an even lower value will exist behind it.
According to equation (15)(or(16)) the pressure yields:

$$p_2 = p_0 + \frac{1}{2}\rho[(1-2a)^2 - \beta^2(1-a)^2]V_0^2 = p_0 + \left[\left(\frac{1}{9} - \frac{4}{9}\beta^2\right)\frac{1}{2}\rho V_0^2\right]$$

$$= p_0 - \frac{15}{9}\frac{1}{2}\rho V_0^2 \tag{52}$$

From equation (18) it can however been seen that the classical pressure over the wind turbine remains

$$p_1 - p_2 = \frac{1}{2}\rho[4a(1-a)]V_0^2 = \frac{8}{9}\frac{1}{2}\rho V_0^2 \tag{53}$$

The velocity at the wind turbine exit is, according to the assumptions, equal to:

$$V_3 = (1-a)V_0 = \frac{2}{3}V_0 \tag{54}$$

and the pressure at the diffuser exit was already derived above:

$$p_3 = p_0 - \frac{1}{3}\frac{1}{2}\rho V_0^2 \tag{55}$$

Thus even when the empty diffuser does not exhibit a (negative) back pressure, it will be found behind the DAWT operating with a wind turbine rotor repelling the flow.
The situation including a back pressure ratio larger than 1 follows in a similar way. Assuming a rather arbitrary value γ=1.2 the velocity at the wind turbine inside the diffuser yields:

$$V_1 = \frac{8}{3}V_0 = 1.6V_0 \tag{56}$$

The pressure in front of the diffuser equals:

$$p_1 = p_0 + \rho\left[1 - 2^2(1.2)^2\left(\frac{2}{3}\right)^2\right]V_0^2 = p_0 - 1.56\frac{1}{2}V_0^2 \tag{57}$$

And the pressure just aft of the turbine:

$$p_2 = p_0 + \left[\left(\frac{1}{9} - \frac{4}{9}\beta^2\gamma^2\right)\frac{1}{2}\rho V_0^2\right] = p_0 + \frac{1-2^2(1.2)^2 4}{9}\frac{1}{2}\rho V_0^2$$

$$\approx p_0 - 2.45\frac{1}{2}\,\rho V_0^2 \tag{58}$$

The velocity at the wind turbine exit is in this situation:

$$V_3 = \frac{2.4}{3}V_0 = 0.8V_0 \tag{59}$$

With a corresponding pressure:

$$p_3 = p_0 + \frac{1}{2}\frac{1}{9}\rho V_0^2 - 0.64\frac{1}{2}\,\rho V_0^2 \approx p_0 + 0.53\frac{1}{2}\,\rho V_0^2 \tag{60}$$

from the example it can be seen that also the values of the pressure before an after the wind turbine change when an extra back pressure is assumed at the exit of the DAWT.

At first view it seems strange that these pressures change when part of the diffuser exit may be taken away (effectively shortening the diffuser) without penalty in performance (this is how the establishment of an extra back pressure can be envisaged). The reason for it is that assuming γ > 1 also implies a diffuser with a smaller exit area. Keeping the diffuser area β equal to 2 thus implies a more effective diffuser in terms of its actual area ratio.

It can also been seen from the evaluation of Cp_{rotor} (from equation (20)). This value representing the maximum achievable power based upon the rotor diameter, it is also increased with a factory γ with respect to the first example.

A third numerical example, in which the assumptions β=10/6 and γ=1.2 are made, clarifies the situation. Now the pressure before and after the turbine are again equal to the values from the first example; the value for Cp_{rotor} stays the same but the value for $Cp_{exit}$is increased with the factory γ.

6. Conclusion

A theoretical analysis of both a conventional bare turbine and a DAWT has been undertaken. Unlike the conventional bare turbine case, where a theoretical limit was derived using energy and momentum considerations, the inability to determine the pressure field on the exterior surface of a DAWT meant that a similar limit could not be defined. It was shown that the often used assumption of independence between exit pressure and velocity speed-up or disc loading was incorrect.

MODELING AND DYNAMIC ANALYSIS

Rather than make similar assumptions, geometric, an analysis of the relationship between geometric, operational and flow parameters is presented. It is evident from this that the diffuser efficiency and the exist pressure have the most significant impact upon DAWT augmentation. This therefore provides a clear path for development of DAWT geometries, the focus of which should be to maximize the diffuser efficiency along with the sub-atmospheric pressure at the diffuser exit. It was also apparent that an optimal exit-area-ratio would depend upon economic arguments however, it appears unlikely that the exit-area-ratio would exceed a value around 3.

Nomenclature

A	**Area**	R_e	**Reynolds number**
a	Axial flow interference factor	T	Torque
Ct_{∞}	$\frac{\Delta P_{2-3}}{q\infty}$	V	Velocity
Ct_2	Disc loading coefficient relative to the local dynamic pressure $\frac{\Delta P_{2-3}}{q\infty}$	W	Resultant velocity
Cp_{air}	Available power coefficient $\frac{\Delta P_{2-3} A_r V_2}{\frac{1}{2}\rho V_{\infty}^3 A_r}$	X	Distance in axial direction from DAWT inlet
Cp_4	Exit pressure coefficient $\frac{\Delta P_{\infty-4}}{q\infty}$	Z	Height above ground
D	Drag force	ε	Velocity speed-up $\frac{V_2}{V_{\infty}}$
F	Force	η	Efficiency
g	Acceleration due to gravity	λ	Reciprocal of the diffuser exit-area ratio $\frac{1}{\frac{A_4}{A_3}}$
H	Energy	μ	Dynamic viscosity
$\dot{m}$	Mass-flow rate	σ	Rotor solidity
P	Power	ρ	Density
p	Pressure	ω	Rotational speed of rotor
Q	Volume flow rate	DAWT	Diffuser Augmented Wind Turbine
r_{air}	Available augmentation $\frac{Cp_{air}}{.593}$	EAR	Exit-Area Ratio $\frac{A_4}{A_3}$
r_{sh}	Shaft augmentation $\frac{Cp_{sh}}{.593}$	HAWT	Horizontal-Axis Wind Turbine

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