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A TREATMENT OF A SINGULAR SYSTEM OF RELATIVISTIC CHARGED PARTICLE IN AN EXTERNAL ELECTROMAGNETIC FIELD AS CONTINUOUS SYSTEM

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معالجة النظام الأحادي لجسيم نسبي مشحون موجود في مجال كهرومغناطيسي خارجي كنظام متصل

ملخص تم دراسة النظام الأحادي لجسيم مشحون يتحرك في مجال كهرومغناطيسي خارجي باعتباره نظام متصل بالمعالجة اللاجرانجية .
نتيجة البحث تتفق مع النتائج التي تم التوصل إليها بإستخدام المعالجة الهاملتونية .

ABSTRACT The singular system of a relativistic charged particle moving in electromagnetic field is treated as field system in the Lagrangian formulation. The result is in a complete agreement with that results obtained by using the Hamiltonian formulations.

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1. Introduction

The Hamiltonian formulation of singular system was initiated by Dirac [1]. He obtained the equations of motion using the consistency condition. Besides, he showed that the number of degrees of freedom of the dynamical system can be reduced. Recently, the Hamilton-Jacobi formulation has been developed to investigate singular system [2,3,4]. The equations of motion are written as total differential equations in many variables

$$dq_a = \frac{\partial H'_\alpha}{\partial p_a} dq_\alpha \quad , \quad dp_a = -\frac{\partial H'_\alpha}{\partial q_a} dq_\alpha \quad , \quad dp_\mu = -\frac{\partial H'_\nu}{\partial x_\mu} dx_\nu . \qquad (1)$$

$$a = 1,\dots,n-p \quad , \quad \mu,\nu = 0,1,\dots,p$$

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(Einstein's summation rule is used through out this paper). The constraints are written as

$$H'_{\mu} = H_{\mu} + P_{\mu} = 0, \tag{2}$$

where p<n, and n-p is the rank of the Hessian matrix

$$\frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j}, \qquad i, j = 1,...,n. \tag{3}$$

Simultaneous solutions of equation (1) and (2) determine the degree of freedom of the singular system. This degree of freedom is at most p, depending on the integrability conditions of the equation. Solution gives the field in terms of the independent coordinates.
In Ref. [5], the singular system was treated as continuous system. The Euler-Lagrange equations of a singular system are proposed in the form

$$\frac{\partial}{\partial x_{\mu}}\left[\frac{\partial L'}{\partial_{\mu} q_a}\right] - \frac{\partial L'}{\partial q_a} = 0,$$
$$a = 1,..,n-p \quad , \quad \mu = n-p+1,...,n, \tag{4}$$

with constraints,

$$dG_{\mu} = -\frac{\partial L'}{\partial x_{\mu}} dt, \tag{5}$$

where L' is the modified Lagrangian which is written as

$$L'(x_{\mu}, q_a, \partial_{\mu} q_a, \dot{x}_{\mu}) \equiv L(x_{\mu}, q_a, \dot{q}_a = (\partial_{\mu} q_a)\dot{x}_{\mu}), \qquad \dot{x}_{\mu} = \frac{dx_{\mu}}{dt}. \tag{6}$$

and

$$G_\mu = H_\mu\left[x_\mu, q_a, p_a = \frac{\partial L}{\partial \dot q_a}\right]. \qquad (7)$$

The variation of constraint (7) should be considered in order to have a consistent theory.

Our aim in this paper is to study the motion of relativistic particle in an external electromagnetic field. We will treat this singular system as continuous or field system.

2. Canonical Formulation of Relativistic Particle in An External Electromagnetic Field

The motion of relativistic particle of charge (e) in an external electromagnetic field is described by the first order singular Lagrangian [6,7]

$$L = -mc\left[g^{\alpha\beta}\dot q_\alpha \dot q_\beta\right]^{1/2} - \frac{e}{c}\dot q_\alpha A_\alpha . \qquad \alpha,\beta = 0,1.2,3 \qquad (8)$$ He

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$$\dot q_\alpha = \frac{dq_\alpha}{d\tau}, \qquad (9)$$ w

here τ is a parameter and. $g_{00} = 1, g_{11} = g_{22} = g_{33} = -1$

The rank of the Hessian matrix

$$\frac{\partial^2 L}{\partial \dot q_\alpha \partial \dot q_\beta}, \qquad \alpha,\beta = 0,1,2,3 \qquad (10)$$

is three, the momenta canonically conjugate to $\dot q_\alpha$ are

$$p_0 = -mc\dot q_0\left[\dot q_0 - \dot q_1 - \dot q - \dot q_3\right]^{-1/} - \frac{e}{c}A_0, \qquad (11)$$

$$p_a = mc\dot q_a\left[\dot q_0 - \dot q_1 - \dot q - \dot q_3\right]^{-1/2} + \frac{e}{c}A_a. \qquad a = 1,2,3 \qquad (12)$$

Since the rank of the Hessian matrix is three, one can solve equation (12) to obtain the generalized velocities $\dot{q}_a$ in term of p_a ،$\dot{q}_0$ and A_a as

$$\dot{q}_a = \frac{\pm \dot{q}_0 \left(\mathbf{p} - (e/c)\mathbf{A}\right)_a}{\left[\left|\mathbf{p} - (e/c)\mathbf{A}\right| \; + m \; c \;\right]^{1/2}} \equiv W_a \qquad (13)$$

Substituting equation (13) into equation (11), we get

$$p \;\; = -\left[\left|\mathbf{p} - (e/c)\mathbf{A}\right| \; + m \; c \;\right]^{1/2} - \frac{e}{c} A_0 = -H_0 \qquad (14)$$

Equation (14) represents the constraint equation,

$$H_0' = p_0 + \frac{e}{c} A_0 + \left[\left|\mathbf{p} - (e/c)\mathbf{A}\right| \; + m \; c \;\right]^{1/2} = 0. \qquad (15)$$

The canonical Hamiltonian H corresponding to τ is written as

$$H = p_a w_a + p_0 \dot{q}_0 - L,$$

$$H = q_0 \left\{ \frac{\left|\mathbf{p} - (e/c)\mathbf{A}\right| \; + m\, c}{\left[\left|\mathbf{p} - (e/c)\mathbf{A}\right| \; + m\, c\right]^{1/}} - \left[\left|\mathbf{p} - (e/c)\mathbf{A}\right| \; + m\, c\right]^{1/2} \right\} \equiv 0, \qquad (16)$$

which leads to

$$H' = \pi + H = 0, \qquad (17)$$

where π is the momenta corresponding to τ .

The equations of motion are the total differential equations:

$$dq_a = \frac{\partial H'}{\partial p_a} d\tau + \frac{\partial H'}{\partial p_a} dq \quad , \quad p_a = -\frac{\partial H'}{\partial q_a} d\tau - \frac{\partial H'}{\partial q_a} dq \;\; , \qquad (18)$$

$$dp \;\; = -\frac{\partial H'}{\partial q} d\tau - \frac{\partial H'}{\partial q} dq \quad , \quad d\pi = -\frac{\partial H'}{\partial \tau} d\tau - \frac{\partial H'}{\partial \tau} dq_0 . \qquad (19)$$

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Using (15) and (17), the equations of motion are reduced to the form

$$dq_a = \frac{\partial H^{'}}{\partial p_a} dq = \frac{(\mathbf{p}-(e/c)\mathbf{A})_a}{\left[\left|\mathbf{p}-(e/c)\mathbf{A}\right| + m\ c\right]^{1/}} dq \ , \qquad (20)$$

$$dp_a = -\frac{\partial H^{'}}{\partial q_a} dq = \frac{e}{c}\left[\frac{(\mathbf{p}-(e/c)\mathbf{A}).(\partial \mathbf{A}/\partial q_a)}{\left[\left|\mathbf{p}-(e/c)\mathbf{A}\right| + m\ c\right]^{1/2}} - \frac{\partial A_0}{\partial q_a}\right] dq_0, \qquad (21)$$

$$dp = -\frac{\partial H^{'}}{\partial q} dq = \frac{e}{c}\left[\frac{(\mathbf{p}-(e/c)\mathbf{A}).(\partial \mathbf{A}/\partial q\ )}{\left[\left|\mathbf{p}-(e/c)\mathbf{A}\right|^2 + m^2c^2\right]^{1/2}} - \frac{\partial A}{\partial q}\right] dq \ , \qquad (22)$$

$$d\pi = -\frac{\partial H^{'}}{\partial \tau} dq_0 = 0. \qquad (23)$$

The above equations are integrable iff the variations of $H^{'}$ and $H_0^{'}$ vanish identically. It is clear that the variation of $H^{'}$ is identically zero, and

$$dH^{'} = dp + dH = \frac{(e/c)(\partial \mathbf{A}/\partial q\ ).(\mathbf{p}-(e/c)\mathbf{A})}{\left[\left|\mathbf{p}-(e/c)\mathbf{A}\right| + m\ c\right]^{1/}} dq - \frac{e}{c}\frac{\partial A}{\partial q} dq$$

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$$+ d\left[\left|\mathbf{p}-(e/c)\mathbf{A}\right| + m\ c\right]^{1/2} + \frac{e}{c} dA_0 = 0. \qquad (24)$$

ince the variation of H_0 is also identically zero, the equations (20-22) are integrable.

To illustrate this approach, we consider the following two cases:

CASE (1) **Motion in A Uniform Electric Field**

Consider the motion of charge (e) in a uniform electric field,

$$E = E\hat{i}, \qquad (25)$$

where E is a constant. Thus, the only non-zero components of $\vec{A}$ is $A_0 = \phi$, where ϕ is electrostatic potential, and $q_0 \equiv t, q_1 \equiv x, q_2 \equiv y, q_3 \equiv z$ Equations (21) take the forms

$$dp_x = -\frac{e}{c}\frac{\partial \phi}{\partial x} dt = \frac{e}{c} E dt, \tag{26}$$

$$dp_y = 0, \tag{27}$$

$$dp_z = 0, \tag{28}$$

which have the following solutions

$$p_x = \frac{e}{c} Et + c_1, \tag{29}$$

$$p_y = c_2, \tag{30}$$

$$p_z = c_{3,} \tag{31}$$

where c_1, c_2 and c_3 are constants.

Equations of motion (20) take the form

$$dx = \frac{p_x}{\left[|\mathbf{p}| \;\; + m \;\; c \;\;\right]^{1/}} dt, \tag{32}$$

$$dy = \frac{p_y}{\left[|\mathbf{p}| \;\; + m \;\; c \;\;\right]^{1/}} dt, \tag{33}$$

$$dz = \frac{p_z}{\left[|\mathbf{p}| \;\; + m \;\; c \;\;\right]^{1/2}} dt. \tag{34}$$

To simplify the problem, let. $c_1 = c_3 = 0$ This leads to the solution

$$x = \frac{c}{eE}\left[\left(\frac{e}{c} Et\right) \;\; + c \;\; + m \;\; c \;\;\right]^{/2}, \tag{35}$$

$$y = \frac{c_2}{(e/c)E} \sinh^{-1}\left(\frac{eE}{ck} t\right), \tag{36}$$

where

$$k = c_2^2 + m^2 c^2. \tag{37}$$

CASE (2) Motion in A Uniform Magnetic Field

Consider a constant magnetic field in the negative z-direction, where the components of the vector field $\vec{A}$ are

$$A_x = -\frac{1}{2}Hy, \quad A_y = \frac{1}{2}Hx, \quad A_0 = A_z = 0. \tag{38}$$

Equation (20) and (21) take the forms

$$dx = \frac{p_x - (e/c)A_x}{k}dt = \frac{p_x + (eH/2c)y}{k}dt, \tag{39}$$

$$dy = \frac{p_y - (e/c)A_y}{k}dt = \frac{p_y - (eH/2c)x}{k}dt, \tag{40}$$

$$dz = \frac{p_z - (e/c)A_z}{k}dt = \frac{p_z}{k}dt, \tag{41}$$

$$dp_x = \frac{(e/c)(\partial A_y/\partial x)(p_y - (e/c)A_y)}{k}dt = \frac{(eH/2c)(p_y - (eH/2c)x)}{k}dt, \tag{42}$$

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$$dp_y = \frac{(e/c)(\partial A_x/\partial y)(p_x - (e/c)A_x)}{k}dt = \frac{-(eH/2c)(p_x + (eH/2c)y)}{k}dt, \tag{43}$$

$$dp_z = 0, \tag{44}$$

here

$$k = \left[\left|\mathbf{p} - \frac{e}{c}\mathbf{A}\right| + m\ c\right]^{1/2}. \tag{45}$$

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quation (40) and (42) lead to

$$\frac{dp_x}{dt} = \dot{p}_x = \frac{eH}{2c}\dot{y}, \tag{46}$$

Similarly, equation (39) and (43) lead to

$$\frac{dp_y}{dt} = \dot{p}_y = -\frac{eH}{2c}\dot{x}. \tag{47}$$

3.Treatment of A singular System of Relativistic Particle in an External Electromagnetic Field as Continuous System

Now, let us attack the problem by using Lagrangian formulation. Since the rank of the Hessian matrix is three, we can treat the problem as continuous system by expressing the variables $q_{1,}q_2$ and q_3 as

$$q_1 = q_1(q_0, \tau) \quad , \quad q_2 = q_2(q_0, \tau) \quad , \quad q_3 = q_3(q_0, \tau). \qquad (48)$$

Thus, $\dot{q}_1, \dot{q}_2$ and $\dot{q}_3$ are written as

$$\dot{q}_1 = \frac{\partial q_1}{\partial \tau} + \frac{\partial q_1}{\partial q_0}\dot{q}_0, \qquad \dot{q}_2 = \frac{\partial q_2}{\partial \tau} + \frac{\partial q_2}{\partial q_0}\dot{q}_0, \qquad \dot{q}_3 = \frac{\partial q_3}{\partial \tau} + \frac{\partial q_3}{\partial q_0}\dot{q}_0 . (49)$$

Euler-Lagrange equation takes the form

$$\frac{\partial}{\partial x_\mu}\left[\frac{\partial L'}{\partial_\mu q_a}\right] - \frac{\partial L'}{\partial q_a} = 0,$$

$$x_\mu \equiv q_0, \tau \quad , \quad q_a \equiv q_1, q_2, q_3 \qquad (50)$$

where the modified Lagrangian "L'" is given by

$$L' = -mc\left[\dot{q} - \left(\frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial q}\dot{q}\right) - \left(\frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial q}\dot{q}\right) - \left(\frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial q}\dot{q}\right)\right]^{/2} - \frac{e}{c}\dot{q}_0 A_0$$

$$+\frac{e}{c}\left(\frac{\partial q_1}{\partial \tau} + \frac{\partial q_1}{\partial q_0}\dot{q}_0\right)A_1 + \frac{e}{c}\left(\frac{\partial q_2}{\partial \tau} + \frac{\partial q_2}{\partial q_0}\dot{q}_0\right)A_2 + \frac{e}{c}\left(\frac{\partial q_3}{\partial \tau} + \frac{\partial q_3}{\partial q_0}\dot{q}_0\right)A_3, \qquad (51)$$

Explicitly, equation (50) becomes

where,

$$\frac{\partial}{\partial \tau}\left\{mc\left(\frac{\partial q_a}{\partial \tau} + \frac{\partial q_a}{\partial q_0}\dot{q}_0\right)B + \frac{e}{c}A_a\right\} + \frac{\partial}{\partial q_0}\left\{mc\left(\frac{\partial q_a}{\partial \tau} + \frac{\partial q_a}{\partial q_0}\dot{q}_0\right)B + \frac{e}{c}A_a\right\}\dot{q}_0 + \frac{e}{c}\dot{q}_0\frac{\partial A_0}{\partial q_a}$$

$$-\frac{e}{c}\left(\frac{\partial q_1}{\partial \tau} + \frac{\partial q_1}{\partial q_0}\dot{q}_0\right)\frac{\partial A_1}{\partial q_a} - \frac{e}{c}\left(\frac{\partial q_2}{\partial \tau} + \frac{\partial q_2}{\partial q_0}\dot{q}_0\right)\frac{\partial A_2}{\partial q_a} - \frac{e}{c}\left(\frac{\partial q_3}{\partial \tau} + \frac{\partial q_3}{\partial q_0}\dot{q}_0\right)\frac{\partial A_3}{\partial q_a} = 0, a = 1,2,3 \quad (52)$$

From the constraint equation (5,7),

$$dG = \frac{\partial L'}{\partial \tau} d\tau, \tag{54}$$

$$G = H\left[q_a, x_\mu, p_a = \frac{\partial L}{\partial \dot{q}_a} \right] = 0, \tag{55}$$

where, H is the Hamiltonian corresponding to the parameter τ , one gets

$$\frac{\partial L'}{\partial \tau} = 0. \tag{56}$$

It is clear that L' depends only on q_0, furthermore q_a also function of q_0 .The vanishing of the Hamiltonian corresponding to τ enables us to treat q_0 as τ .Thus equation (52) reduces to the form

$$\frac{d}{dq}\left\{ mc\frac{dq_a}{dq}B' + \frac{e}{c}A_a \right\} + \frac{e}{c}\frac{\partial A_0}{\partial q_a} - \frac{e}{c}\frac{dq_1}{dq_0}\frac{\partial A_1}{\partial q_a} - \frac{e}{c}\frac{dq_2}{dq_0}\frac{\partial A_2}{\partial q_a} - \frac{e}{c}\frac{dq_3}{dq_0}\frac{\partial A_3}{\partial q_a} = 0, \qquad a = 1,2,3 \tag{57}$$

where,

$$B' = \left[1 - \left(\frac{dq_1}{dq_0} \right) - \left(\frac{dq}{dq_0} \right) - \left(\frac{dq_3}{dq_0} \right) \right]^{-1/2}. \tag{58}$$

To check the validity of this section, let us consider the following two simple cases which have been discussed in section 2 :

CASE (1) **Motion in A Uniform Electric Field**

Consider the motion of charge (e) in a uniform electric field.

$$E = E\hat{i}, \tag{59}$$

where, E is constant. The only non-zero components of $\vec{A}$ is $A_0 = \phi$,where ϕ is the electrostatic potential, and $q_0 \equiv t$ ، $q_1 \equiv x, q_2 \equiv y, q_3 \equiv z$ .Thus, equations(57) are written as

$$\frac{d}{dt}\left\{mc\frac{dx}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/}\right\}=-\frac{e}{c}\frac{\partial\phi}{\partial x}=\frac{e}{c}E, \tag{60}$$

$$\frac{d}{dt}\left\{mc\frac{dy}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/}\right\}=0, \tag{61}$$

$$\frac{d}{dt}\left\{mc\frac{dz}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/2}\right\}=0, \tag{62}$$

which, have the following solutions

$$mc\frac{dx}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/}=\frac{e}{c}Et+c_1, \tag{63}$$

$$mc\frac{dy}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/2}=c_2, \tag{64}$$

$$mc\frac{dz}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/2}=c_3, \tag{65}$$ w

here, c_1, c_2 and c_3 are constants.

Equations (63),(64) and 65) can be written as

$$\frac{dx}{dt}=\frac{(e/c)Et}{\left[\left[\frac{e}{c}Et\right]+c\;+m\;c\right]^{1/}}, \tag{66}$$

$$\frac{dy}{dt}=\frac{c}{\left[\left[\frac{e}{c}Et\right]+c\;+m\;c\right]^{1/2}}, \tag{67}$$

where, c_1 and c are chosen to be zeros.

The solutions of (66) and (67) respectively, are

$$x = \frac{c}{eE}\left[\left(\frac{e}{c}Et\right) + c \;\; + m \;\; c \;\;\right]^{/2}, \tag{68}$$

$$y = \frac{c_2}{(e/c)E}\sinh^{-1}\left(\frac{eE}{ck}t\right), \tag{69}$$

where

$$k = c_2^2 + m^2c^2. \tag{70}$$

CASE(2) **Motion in A Uniform Magnetic Field**

Consider a constant magnetic field in the negative z-direction, such that the components of the vector field $\vec{A}$ are

$$A_x = -\frac{1}{2}Hy, \quad A_y = \frac{1}{2}Hx, \quad A_0 = A_z = 0, \tag{71}$$

and . $q_1 \equiv x, q_2 \equiv y, q_3 \equiv z$ Equation (57) take the following forms:

$$\frac{d}{dt}\left\{mc\frac{dx}{dt}\left[1-\left(\frac{dx}{dt}\right) - \left(\frac{dy}{dt}\right) - \left(\frac{dz}{dt}\right)\right]^{-1/2} - \frac{eH}{2c}y\right\} = \frac{eH}{2c}\frac{dy}{dt}, \tag{72}$$

$$\frac{d}{dt}\left\{mc\frac{dy}{dt}\left[1-\left(\frac{dx}{dt}\right) - \left(\frac{dy}{dt}\right) - \left(\frac{dz}{dt}\right)\right]^{-1/} + \frac{eH}{2c}x\right\} = -\frac{eH}{2c}\frac{dx}{dt}, \tag{73}$$

and

$$\frac{d}{dt}\left\{mc\frac{dz}{dt}\left[1-\left(\frac{dx}{dt}\right) - \left(\frac{dy}{dt}\right) - \left(\frac{dz}{dt}\right)\right]^{-1/}\right\} = 0. \tag{74}$$

where

$$mc\frac{dx}{dt}\left[1-\left(\frac{dx}{dt}\right) - \left(\frac{dy}{dt}\right) - \left(\frac{dz}{dt}\right)\right]^{-1/2} - \frac{eH}{2c}y = p_x, \tag{75}$$

$$mc\frac{dy}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/}+\frac{eH}{2c}x=p_y, \qquad (76)$$

and,

$$mc\frac{dz}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/2}=p_z, \qquad (77)$$

Equations (72) and (73) are in a complete agreement with equations (46) and (47) respectively.

Integrating equation (72) and (73), we get

$$mc\frac{dx}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/}=\frac{eH}{c}y+A; \qquad (78)$$

$$mc\frac{dy}{dt}\left[1-\left(\frac{dx}{dt}\right)-\left(\frac{dy}{dt}\right)-\left(\frac{dz}{dt}\right)\right]^{-1/2}=-\frac{eH}{c}x+B, \qquad (79)$$

Solving equations (79) and (78) with A=B=0,we get

$$\frac{eH}{c}y\frac{dy}{dt}+\frac{eH}{c}x\frac{dx}{dt}=0. \qquad (80)$$

Integration of equation (80) leads to

$$\frac{y^2}{(2c/eH))}+\frac{x^2}{(2c/eH))}=C, \qquad (81)$$

where C is constant .Besides, one gets

$$p_z=const, \qquad v_z=const \qquad (82)$$

From equation (81) and (82), we observe that the charged particle moves along a helix.

Conclusion

The motion of a relativistic charged particle in an external electromagnetic field is a standard procedure. Since the Lagrangian is singular, this system has been investigated as singular system [6,7]. The system can be investigated by two methods, Dirac's and the canonical approach .

In this work, we use a direct method to study the Lagrangian formulation of singular systems. The system of charged particle moving in electromagnetic field is treated as continuous system, and Euler-Lagrange equations are solved with some constraints.

The worth point of this treatment is both Lagrangian and Hamiltonian formulations are used together. Also in both the canonical method and the Lagrangian formulation as field system, there is no need to use the gauge fixing conditions. Whereas in Dirac's approach the gauge fixing is necessary to determine the Lagrange multiplier λ, which is an arbitrary function. This type of arbitrary functions is inevitable and they should be determined by imposing new gauge conditions, which is not easy task in Dirac's method.

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