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Peristaltic Transport of a Magneto-Newtonian Fluid Through a Porous Medium

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Abstract: Peristaltic motion of an incompressible Newtonian fluid through a porous medium is studied in a two-dimensional uniform channel with a sinusoidal wave on its wall. The fluid is subjected to constant transverse magnetic field. Using a perturbation expansion on the wave number as a parameter, the velocity field and the pressure gradient are obtained in explicit form. Moreover, the pressure rise and the friction force were computed numerically. The results show that the pressure rise increases as the permeability decreases and it increases as the magnetic parameter increases. Further, it is noted that both pressure rise and friction force does not depend on permeability parameter as well as magnetic parameter at a certain value of flow rate. The results were studied for various values of the physical parameters of interest.

Key Words: peristalsis, Newtonian fluid, porous medium, magnetic field.

1. Introduction

Investigation of flow dynamics of a fluid in a channel, induced by a wave traveling on its wall, has many applications in various branches of science. The physical mechanism of the flow induced by the traveling wave can be understood and is known as the so-called peristaltic transport mechanism. The analysis of the mechanism responsible for peristaltic transport has been studied by many authors. Latham's investigation [10] may be the first study in this field and since that time several theoretical and experimental investigations have been made to understand peristaltic action in both mechanical and physiological situations. Some of these studies were made by Burns and Parkes [4], Barton and Raynor [2], Shapiro *et al*.[17], Roos and Lykoudis [16], Shukla *et al*. [18], Srivastava *et al*. [22,23,24], Srivastava and Srivastava [20,21], Srivastava [19], Elshehawey and Mekheimer [8], Bohme and Friedrich [3], El Misery *et al*. [6], Elshehawey *et al*.[7], Mekheimer *et al*. [11] and Elshehawey and Sobh [9] .

The fluid motion through a porous medium has been studied by many authors. The usual starting point in the solution of problems is the Darcy's experimental law. Some studies about this point have been made by Varshney [26], Raptis *et al*. [13,15], Raptis and Peridikis [14], and El-Dabe and El-Mohandis [5].

Many authors have studied the magnetohydrodynamic flow. Mitra and Bhattacharyya [12] studied the magnetohydrodynamic flow of the fluid suspension, while Sud *et al*. [25] investigated the effect of moving magnetic field on blood flow. They observed that the effect of suitable moving magnetic field accelerate the speed of blood.

With the above discussion in mind we purpose to study the effect of porous medium as well as the transverse magnetic on peristaltic motion of a Newtonian fluid. This model may be applied to the movement of the urine in the ureter in the presence of some stones in its lumen. Because of the complexity of the governing equations, a perturbation solution with wave number as a parameter is obtained to first order. The velocity field and pressure gradient were obtained in explicit forms. Moreover, the pressure rise per unit wavelength and the friction force were computed numerically and were plotted with the variation of the flow rate.

2. Formulation and Analysis

We shall consider a two-dimensional channel of uniform thickness 2a, filled with an incompressible conducting Newtonian fluid through a porous medium occupying a semi-infinite region of the space and is subjected to constant transverse magnetic field. The walls of the channel are flexible and non-conducting, on which are imposed travelling sinusoidal waves of moderate amplitude. The geometry of the wall surface is

$$\overline{H}(\overline{X},\bar{t}) = a + b\sin\frac{2\pi}{\lambda}(\overline{X} - c\bar{t}), \qquad (2.1)$$

where b is the wave amplitude, λ is the wave length, c is the propagation velocity, , $\bar{t}$ is the time and $\overline{X}$ is the same direction of the wave propagation.

Choosing moving coordinates $(\bar{x},\bar{y})$, (wave frame), which travel in the $\overline{X}$-direction with the same speed as the wave, the unsteady flow in the laboratory frame $(\overline{X},\overline{Y})$ can be treated as steady [17]. The coordinates frame are related through

$$\bar{x} = \overline{X} - c\bar{t}, \qquad \bar{y} = \overline{Y}, \qquad (2.2)$$

$$\bar{u} = \overline{U} - c, \qquad \bar{v} = \overline{V}, \qquad (2.3)$$

where $(\overline{U},\overline{V})$ and $(\bar{u},\bar{v})$ are the velocity components in the corresponding coordinate system.

The non-dimensional forms of equation of continuity, equations of motion and the boundary conditions, respectively, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (2.4)$$

$$Re\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\left(\frac{1}{k}+M\,\mathrm{Re}\right)u \qquad (2.5)$$

$$Re\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\delta^2\frac{v}{k}, \qquad (2.6)$$

$$\frac{\partial u}{\partial y}=0, \qquad v=0 \qquad \text{for} \quad y=0, \qquad (2.7a)$$

$$u=-1, \qquad v=-\frac{dH}{dx} \qquad \text{for} \quad y=H, \qquad (2.7b)$$

where the non-dimensional variables and parameters,

$$x=\frac{\bar{x}}{\lambda},X=\frac{\bar{X}}{\lambda},y=\frac{\bar{y}}{a},Y=\frac{\bar{Y}}{a},t=\frac{c\bar{t}}{\lambda},p=\frac{a^2\bar{p}}{c\lambda\mu},v=\frac{\lambda\bar{v}}{ac},u=\frac{\bar{u}}{c},\ \varphi=\frac{b}{a}$$

$$U=\frac{\bar{U}}{c},\delta=\frac{a}{\lambda},Re=\frac{\rho ca}{\mu},k=\frac{\bar{k}}{a^2},\ M=\frac{\sigma B_0^2 a}{\rho c},$$

$$H=\frac{\bar{H}}{a}=1+\frac{b}{a}\sin 2\pi x=1+\varphi\sin 2\pi x,$$

have been used.

Note that $\bar{p}$ is the pressure, μ is the viscosity, ρ is the density, ν_1 is the kinematic viscosity, $\bar{k}$ is the permeability parameter, B_0 is the constant magnetic field, σ is the electrical conductivity, δ is the wave number, Re is the Reynolds number, M is the magnetic parameter and $\varphi<1$, is the amplitude ratio.

3. Rate of Volume Flow

The instantaneous volume flow rate in the fixed frame is given by

$$Q=\int_0^{\bar{H}}\bar{U}(\bar{X},\bar{Y},\bar{t})\,d\bar{Y}, \qquad (3.1)$$

where $\bar{H}$ is a function of $\bar{X}$ and $\bar{t}$.

The rate of volume flow in the moving frame (wave frame) is given by

$$\bar{q}=\int_0^{\bar{H}}\bar{u}(\bar{x},\bar{y})\,d\bar{y}, \qquad (3.2)$$

where $\bar{H}$ is a function of $\bar{x}$.

Using equation (2.3), one finds that the two rates of volume flow are related by

$$Q=\bar{q}+c\bar{H}. \qquad (3.3)$$

The time-mean flow over a period $T = \frac{\lambda}{c}$ at a fixed position $\overline{X}$ is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T Q \, d\bar{t} \,, \tag{3.4}$$

which can be written, using (2.1) and (3.3), as

$$\overline{Q} = \overline{q} + ac \,. \tag{3.5}$$

Defining the dimensionless time-mean flows Θ and F in the fixed and wave frame respectively as

$$\Theta = \frac{\overline{Q}}{ac} \qquad \text{and} \qquad F = \frac{\overline{q}}{ac} \,, \tag{3.6}$$

then making use of (3.6), equation (3.5) can be rewritten as

$$\Theta = F + 1 \,, \tag{3.7}$$

where

$$F = \int_0^{H(x)} u \, dy \,. \tag{3.8}$$

4. Perturbation Solution

We expand the following quantities in a power series of the small parameter δ as follows

$$u = u_0 + \delta u_1 + O(\delta^2)$$

$$v = v_0 + \delta v_1 + O(\delta^2)$$

$$\frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + \delta \frac{\partial p_1}{\partial x} + O(\delta^2)$$

$$F = F_0 + \delta F_1 + O(\delta^2) \,, \tag{4.1}$$

The use of expansions (4.1) with equations (2.4), (2.5), (2.6) and (2.7) gives the following systems

System of order zero

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \,, \tag{4.2}$$

$$\frac{\partial p_0}{\partial x} = \frac{\partial^2 u_0}{\partial y^2} - \frac{1}{k_1} u_0 \,, \tag{4.3}$$

$$\frac{\partial p_0}{\partial y} = 0 \,, \tag{4.4}$$

with the boundary conditions

$$\frac{\partial u_0}{\partial y} = 0, \qquad v_0 = 0 \qquad \text{for} \qquad y = 0, \qquad (4.5a)$$

$$u_0 = -1, \qquad v_0 = -\frac{dH}{dx} \qquad \text{for} \qquad y = H. \qquad (4.5b)$$

where $k_1 = \dfrac{k}{1 + kM\,\mathrm{Re}}$.

The solution of this system for u_0 subject to the boundary conditions is

$$u_0 = k_1\left(\frac{dp_0}{dx}\right)\left[\cosh\left(\frac{y}{\sqrt{k_1}}\right)\Big/\cosh\left(\frac{H}{\sqrt{k_1}}\right) - 1\right] - \cosh\left(\frac{y}{\sqrt{k_1}}\right)\Big/\cosh\left(\frac{H}{\sqrt{k_1}}\right). \quad (4.6)$$

It is clear that as k tends to infinity and M tends to zero, the velocity u_0 becomes

$$u_0 = \frac{1}{2}\left(\frac{dp_0}{dx}\right)\left[y^2 - H^2\right] - 1.$$

The instantaneous volume flow rate F_0 in the moving coordinates is given by

$$F_0 = \int_0^H u_0\,dy$$

$$= k_1\left(\frac{dp_0}{dx}\right)\left[\sqrt{k_1}\tanh\left(\frac{H}{\sqrt{k_1}}\right) - H\right] - \sqrt{k_1}\tanh\left(\frac{H}{\sqrt{k_1}}\right),$$

which implies that

$$\frac{dp_0}{dx} = \left[F_0 + \sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right)\right]\Big/\left[k^{3/2}\tanh\left(\frac{H}{\sqrt{k_1}}\right) - k_1 H\right]. \quad (4.7)$$

Using (4.2), (4.6) and (4.7), we obtain the alternative form of u_0 and v_0 as

$$u_0 = c_0 + c_1\cosh\left(\frac{y}{\sqrt{k_1}}\right), \qquad (4.8a)$$

$$v_0 = -c_0' y - c_1'\sqrt{k_1}\sinh\left(\frac{y}{\sqrt{k_1}}\right), \qquad (4.8b)$$

where

$$c_0 = -k_1(b_0 F_0 + b_1), \; c_1 = (a_0 F_0 + a_1)$$

$$a_0 = \left[\sqrt{k_1}\sinh\left(\frac{H}{\sqrt{k_1}}\right) - H\cosh\left(\frac{H}{\sqrt{k_1}}\right)\right]^{-1}, \; b_0 = \frac{a_0}{k_1}\cosh\left(\frac{H}{\sqrt{k_1}}\right),$$

$$a_1 = \left[a_0 \sqrt{k_1} \sinh\left(\frac{H}{\sqrt{k_1}} \right) - 1 \right] \Big/ \cosh\left(\frac{H}{\sqrt{k_1}} \right), \; b_1 = \frac{a_0}{\sqrt{k_1}} \sinh\left(\frac{H}{\sqrt{k_1}} \right). \quad (4.9)$$

System of order one

Equating the coefficients of δ on both sides in (2.4), (2.5) and (2.6), we get

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \;, \quad (4.10)$$

$$Re\left(u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right) = -\frac{\partial p_0}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} - \frac{1}{k_1} u_1 , \quad (4.11)$$

$$\frac{\partial p_1}{\partial y} = 0 , \quad (4.12)$$

with the boundary conditions

$$\frac{\partial u_1}{\partial y} = 0, \qquad v_1 = 0 \qquad \text{for} \qquad y = 0 , \quad (4.13a)$$

$$u_1 = 0, \qquad v_1 = 0 \qquad \text{for} \qquad y = H . \quad (4.13b)$$

Solving this system for u_1, after using (4.8), we obtain

$$u_1 = k_1 \left(\frac{dp_1}{dx} \right) \left[\cosh\left(\frac{y}{\sqrt{k_1}} \right) \Big/ \cosh\left(\frac{H}{\sqrt{k_1}} \right) - 1 \right] + \mathrm{Re}\left[-\frac{c_0' c_1}{4} (y^2 - H^2) \cosh\left(\frac{H}{\sqrt{k_1}} \right) \right.$$

$$+ \frac{\sqrt{k_1}}{4} (2 c_0 c_1' + 3 c_0' c_1) \left\{ y \sinh\left(\frac{y}{\sqrt{k_1}} \right) - H \tanh\left(\frac{H}{\sqrt{k_1}} \right) \cosh\left(\frac{y}{\sqrt{k_1}} \right) \right\}$$

$$\left. + k_1 (c_0 c_0' + c_1' c_1) \left\{ \cosh\left(\frac{y}{\sqrt{k_1}} \right) \Big/ \cosh\left(\frac{H}{\sqrt{k_1}} \right) - 1 \right\} \right] . \quad (4.14)$$

The instantaneous volume flow rate F_1 is given by

$$F_1 = \int_0^H u_1 \, dy$$

$$= k_1 \left(\frac{dp_1}{dx} \right) \Big/ \left[a_0 \cosh\left(\frac{H}{\sqrt{k_1}} \right) \right] + \mathrm{Re}\left[k (c_0 c_0' + c_1 c_1') \Big/ \left(a_0 \cosh\left(\frac{H}{\sqrt{k_1}} \right) \right) \right.$$

$$\left. - \frac{c_1 c_0' k_1}{2 a_0} - \frac{k_1}{4} (2 c_0 c_1' + 3 c_0' c_1) \left\{ \frac{1}{a_0} + H \tanh\left(\frac{H}{\sqrt{k_1}} \right) \sinh\left(\frac{H}{\sqrt{k_1}} \right) \right\} \right] .$$

On solving this equation for $\frac{dp_1}{dx}$, one finds

$$\frac{dp_1}{dx} = b_0 F_1 + \mathrm{Re}\left(b_3 F_0^2 + b_4 F_0 + b_5\right), \tag{4.15}$$

where

$$b_2 = -\frac{k_1 b_0}{4}\left\{\frac{1}{a_0} + H \tanh\left(\frac{H}{\sqrt{k_1}}\right)\sinh\left(\frac{H}{\sqrt{k_1}}\right)\right\},$$

$$b_3 = 2k_1 b_0 b_2 a_0' - a_0 a_0' - k_1^2 b_0 b_0' + 3k_1 a_0 b_2 b_0' - \frac{k_1^2 b_0'}{2},$$

$$b_4 = 2k_1 b_1 b_2 a_0' - a_1 a_0' - a_0 a_1' + 2k_1 b_0 b_2 a_1' - \frac{k_1^2 a_1 b_0'}{2a_0} - k_1^2 b_1 b_0'$$
$$+ 3k_1 a_1 b_2 b_0' - \frac{k_1^2 b_1'}{2} - k_1^2 b_0 b_1' + 3k_1 a_0 b_2 b_1',$$

$$b_5 = 2k_1 b_1 b_2 a_1' - a_1 a_1' - k_1^2 b_1 b_1' + 3k_1 a_1 b_2 b_1' - \frac{k_1^2 a_1 b_1'}{2a_0}. \tag{4.16}$$

Using (4.15) in (4.14), the alternative form of u_1 becomes

$$u_1 = c_2 + c_3 \cosh\left(\frac{y}{\sqrt{k_1}}\right)$$
$$+ Re\left[c_4 + c_5 \cosh\left(\frac{y}{\sqrt{k_1}}\right) + c_6 y^2 \cosh\left(\frac{y}{\sqrt{k_1}}\right) + c_7 y \sinh\left(\frac{y}{\sqrt{k_1}}\right)\right], \tag{4.17}$$

where

$$c_2 = -k_1 b_0 F_1\ , \ c_3 = a_0 F_1,\ \ c_4 = -k_1\left(b_3 F_0^2 + b_4 F_0 + b_5 + c_0 c_0' + c_1 c_1'\right),$$

$$c_5 = \frac{k_1\left(b_3 F_0^2 + b_4 F_0 + b_5 + c_0 c_0' + c_1 c_1'\right)}{\cosh\left(\frac{H}{\sqrt{k_1}}\right)} + \frac{c_0' c_1 H^2}{4}$$
$$- \frac{\sqrt{k_1}}{4}\left(2c_0 c_1' + 3c_0' c_1\right) H \tanh\left(\frac{H}{\sqrt{k_1}}\right),$$

$$c_6 = -\frac{c_0' c_1}{4},\ \ c_7 = \frac{\sqrt{k_1}}{4}\left(2c_0 c_1' + 3c_0' c_1\right). \tag{4.18}$$

Now, the axial velocity component u and the pressure gradient $\frac{dp}{dx}$ can be expressed, to first order where $F_0 = F - \delta F_1$, as

$$u = -k_1(b_0 F + b_1) + (a_0 F + a_1)\cosh\left(\frac{y}{\sqrt{k_1}}\right)$$

$$+\delta\, Re\left[a_2 + a_3 \cosh\left(\frac{y}{\sqrt{k_1}}\right) + a_4 y^2 \cosh\left(\frac{y}{\sqrt{k_1}}\right) + a_5\, y \sinh\left(\frac{y}{\sqrt{k_1}}\right)\right], \quad (4.19)$$

$$\frac{dp}{dx} = b_0 F + b_1 + \delta\, Re(b_3 F^2 + b_4 F + b_5), \quad (4.20)$$

where

$$a_2 = -k_1(b_3 + a_0 a_0' + k_1^2 b_0 b_0')F^2 - k_1(b_4 + a_1 a_0' + a_0 a_1' + k_1^2 b_1 b_0' + k_1^2 b_0 b_0')F$$
$$- k_1(b_5 + a_1 a_1' + k_1^2 b_{01} b_1'),$$

$$a_3 = \frac{1}{4\cosh\left(\frac{H}{\sqrt{k}}\right)}\left[\left\{4k\, b_3 + 2k^{3/2}\, b_0 a_0' H \sinh\left(\frac{H}{\sqrt{k_1}}\right) + 4k_1^2 b_0 b_0' + a_0(4a_0' + a_6 b_0')\right\}F^2\right.$$

$$+\left\{4k\, b_4 + k^{3/2}(2b_1 a_0' + 2b_0 a_1' + 3a_0 b_1')H \sinh\left(\frac{H}{\sqrt{k_1}}\right) + 4a_0 a_1' + a_1(4a_0' + a_6 b_0')\right.$$

$$\left. + 4k_1^2(b_1 b_0' + b_0 b_1') - k_1 b_1' a_0 H^2 \cosh\left(\frac{H}{\sqrt{k_1}}\right)\right\}F$$

$$\left. + \left\{4k\, b_5 + 4k\;\, b\, b' + a\;\, a_6 b' + 2k^{3/2} b_1 a_1' H \sinh\left(\frac{H}{\sqrt{k_1}}\right)\right\}\right],$$

$$a_4 = \frac{k_1 a_0 b_0'}{4}F^2 + \frac{k_1}{4}(a_1 b_0' + a_0 b_1')F + \frac{k_1 a_1 b_1'}{4},$$

$$a_5 = -\frac{k^{3/}}{4}(2b\;\, a' + 3a\;\, b')F \;\; - \frac{k^{3/2}}{4}(2b_1 a_0' + 2b_0 a_1' + 3a_1 b_0' + 3a_0 b_1')F$$
$$- \frac{k^{3/2}}{4}(2b_1 a_1' + 3a_1 b_1'),$$

$$a_6 = k_1 H\left[3\sqrt{k_1}\sinh\left(\frac{H}{\sqrt{k_1}}\right) - H\cosh\left(\frac{H}{\sqrt{k_1}}\right)\right]. \quad (4.21)$$

The pressure rise per wavelength Δp_λ and friction force F_λ are given by

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx} dx \,, \qquad (4.22)$$

and

$$F_\lambda = \int_0^1 H\left(-\frac{dp}{dx}\right) dx \,. \qquad (4.23)$$

Using (4.20) in (4.22) and (4.23) then evaluating the integrals numerically we obtain both pressure rise and friction force. The results are discussed through Figs.1-4.

5 Results and Discussion

In our discussion, we have to choose the channel width a and the wave length λ such that the wave number δ be less than one. It is clear that our results calculate the velocity and the pressure gradient in explicit form to the first order of δ without restrictions on the amplitude ratio and the Reynolds number. Further, as M tends to zero and k tends to infinity, the results extend the work of Shapiro *et al.* [17] in which they used two separating expansions, an expansion in powers of δ^2 with Re=0, and an expansion in powers of Re^2 with δ=0.

The relation between pressure rise (Δp_λ) and flow rate (Θ) is displayed in Fig.(1) at Re=10, δ=0.02 , M=10 and $\varphi = 0.8$ with different values of k (k=0.05, 0.1, 0.5 and 10). As shown, the pressure rise does not depend on k at a certain value of flow rate. Also, it is important to note that the pressure rise increases as the permeability decreases. This is because of the resistance caused by the porous medium. In the case of ureter stones this causes renal colic (ureteric colic).

The effect of the magnetic field on the pressure rise is clear in Fig.(2) at Re=10, δ=0.02, k=1, $\varphi = 0.8$ and (M=0, 5, 10 and 15). As shown, the pressure rise increases as the magnetic parameter increases.

Shown in Fig.(3) the effect of porous medium of the friction force at Re=10, δ=0.02, M=10, $\varphi = 0.8$ and (k=0.05, 0.1, 0.5 and 10). It is noted that the friction force increases as the permeability decreases.

In Fig.(4), the relation between friction force and flow rate is plotted at Re=10, δ=0.02, k=1, $\varphi = 0.8$ and (M=0, 5, 10 and 15). It is clear that the friction force increases as the magnetic parameter increases. Also, we notice from Fig.(3) and Fig.(4) that the friction force increases as the flow rate increases.

As a result, explicit forms for the axial velocity component u and the pressure gradient $\frac{dp}{dx}$ for peristaltic motion of a Newtonian fluid in a channel can be obtained from equations (4.19) and (4.20) by letting the permeability parameter k tends to infinity and the magnetic parameter M tends to zero.

References

[1] Ahmadi, G. and Manvi, R. (1971) Equation of motion for viscous flow through a rigid porous medium, Indian J. of Technology, 9, 441-444.

[2] Barton, C. and Raynor, S. (1968) Peristaltic flow in tubes, Bull. Math. Biophys., 30, 663-680.

[3] Bohme, G. B. and Friedrich, R. (1983) Peristaltic flow of viscoelastic liquids, J. Fluid Mech., 128, 109-122.

[4] Burns, J. C. and Parkes, T. (1967) Peristaltic motion, J. Fluid Mech., 29, 731-743.

[5] El-Dabe, N. T. and El-Mohandis, S. (1995) Magnetohydrodynamic flow of second order fluid through a porous medium on an inclined porous plane, The Arabian J. for Sci. and Eng., 20, No. 3, 571-580.

[6] El Misery, A. M., Elshehawey E. F. and Hakeem, A. A. (1996) Peristaltic Motion of an Incompressible Generalized Fluid In a Planar Channel, J. Phys. Soc. Jpn, 65, 501-506.

[7] Elshehawey, E. F., El Misery, A. M. and Hakeem, A. A. (1998) Peristaltic Motion of a Generalized Newtonian Fluid in a Non-Uniform Channel, J. Phys. Soc. Jpn, 67, 434-440.

[8] Elshehawey, E. F. and Mekheimer, K. S. (1994) Couple-Stresses in peristaltic transport of fluids, J. Phys. D: Appl. Phys., 27, 1163-1170.

[9] Elshehawey, E. F. and Sobh, A. M. (2001) Peristaltic viscoelastic fluid motion in a tube, Int. J. of Math. & Math. Sci., 26, No.1, 21-34.

[10] Latham, T. W. (1966) Fluid motion in a peristaltic pump, M. Sc. Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts.

[11] Mekheimer, Kh. S., Elshehawey, E. F. and Elaw, A. M. (1998) Peristaltic motion of a particle-fluid suspension in a planar channel, Int. J. of Theo. Phys., 37, No. 11, 2895-2920.

[12] Mitra, P. and Bhattacharyya, P. (1981) Unsteady hydromagnetic laminar flow of a conducting dusty fluid between two parallel plates started impulsively from rest, Acta Mech., 39, 171

[13] Raptis, A., Kafousias, N. and Massalas, C. (1982) ZAMM, 62, 489-499.

[14] Raptis, A. and Perdiks, C. (1983) Flow of a viscous fluid through a porous medium bounded by a vertical surface, Int. J. Engng. Sci., 21 No. 11, 1327-1336.
[15] Raptis, A., Peridikis, C. and Tzivanidis, G. (1981) Free convection flow through a porous medium bounded by a vertical surface, J. Phys. D: Appl. Phys. 14, L99-108.
[16] Roos, R. and Lykoudis, P. S. (1971) The fluid mechanics of the ureter with an inserted catheter, J. Fluid Mech. 46, 625-630.
[17] Shapiro, A. H., Jaffrin, M. Y. and Weinberg, S. L. (1969) Peristaltic pumping with long wavelength at low Reynolds number, J. Fluid Mech., 37, 799-825.
[18] Shukla, J. B., Parihar, R. S., Rao, R. P. and Gupta, S. P. (1980) Effects of peripheral layer viscosity on peristaltic transport of a bio-fluid, J. Fluid Mech., 97, 255-237.
[19] Srivastava, L. M. (1986) Peristaltic transport of a couple-stress fluid, Rheol. Acta, 25, 638-641.
[20] Srivastava, L. M. and Srivastava, V. P. (1985) Interaction of peristaltic flow with pulsatile flow in a circular cylindrical tube, J. Biomech., 8, 247-253.
[21] Srivastava, L. M. and Srivastava, V. P. (1989) Peristaltic transport of a particle-fluid suspension, J. of Biomech. Engng., 111, 157-165.
[22] Srivastava, L. M., Srivastava, V. P. and Sinha, S. N. (1983) Peristaltic transport of a physiological fluid: part I. Flow in non-uniform geometry, Biorheol., 20, 153-166.
[23] Srivastava, L. M., Srivastava, V. P. and Sinha, S. N. (1983) Peristaltic transport of a physiological fluid: part II. Flow in non-uniform geometry, Biorheol., 20, 167-178.
[24] Srivastava, L. M., Srivastava, V. P. and Sinha, S. N. (1983) Peristaltic transport of a physiological fluid: part III. Flow in non-uniform geometry, Biorheol., 20, 179-186.
[25] Sud, V. K., Sephon, G. S. and Mishra, R. K. (1977) Pumping action on blood flow by amagnetic field, Bulletin Math. Biology, 39, 385.
[26] Varshney, C. L. (1979) The fluctuating flow of a viscous fluid through a porous medium bounded by a porous and horizontal surface, Indian J. Pure Appl. Math., 10, 1558-1568.

Fingers

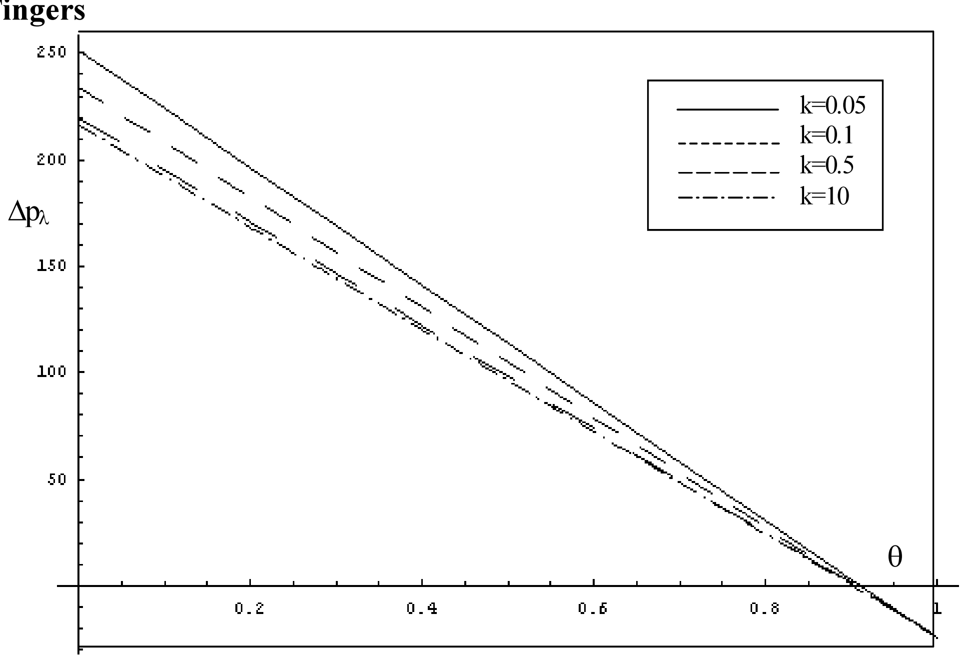

Fig.(1) Pressure rise versus flow rate at Re=10, M=10, φ=0.08 and δ=0.02.

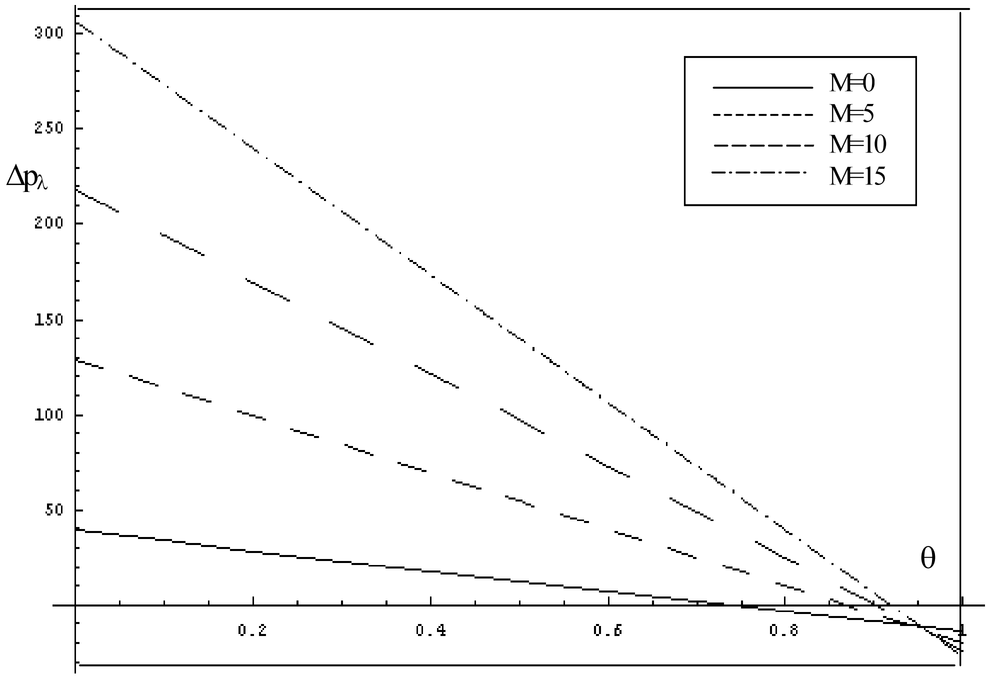

Fig.(2) Pressure rise versus flow rate at Re=10, k=1, φ=0.08 and δ=0.02.

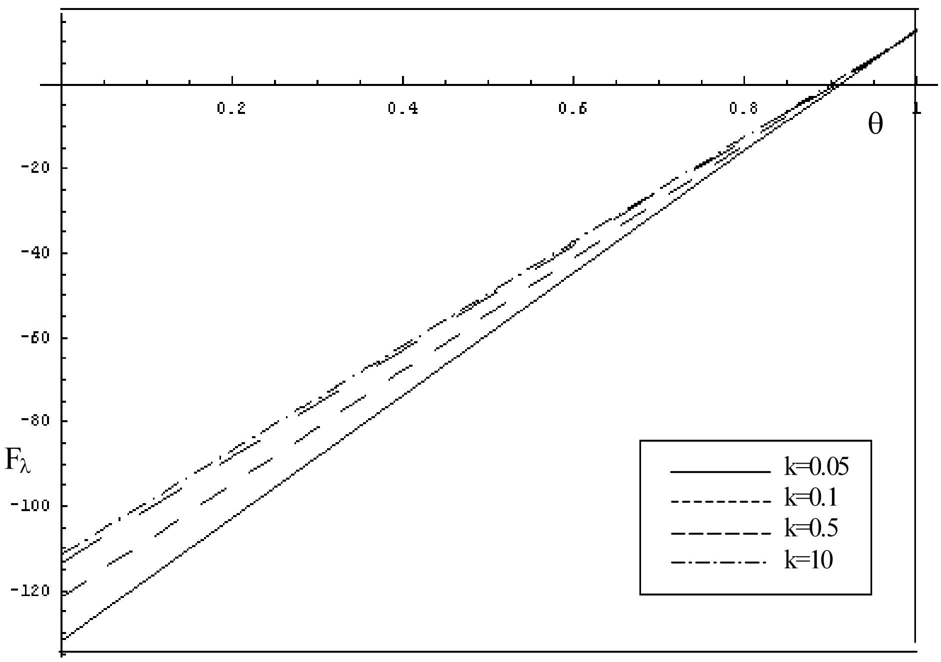

Fig.(3) Friction force versus flow rate at Re=10, M=10, φ=0.08 and δ=0.02.

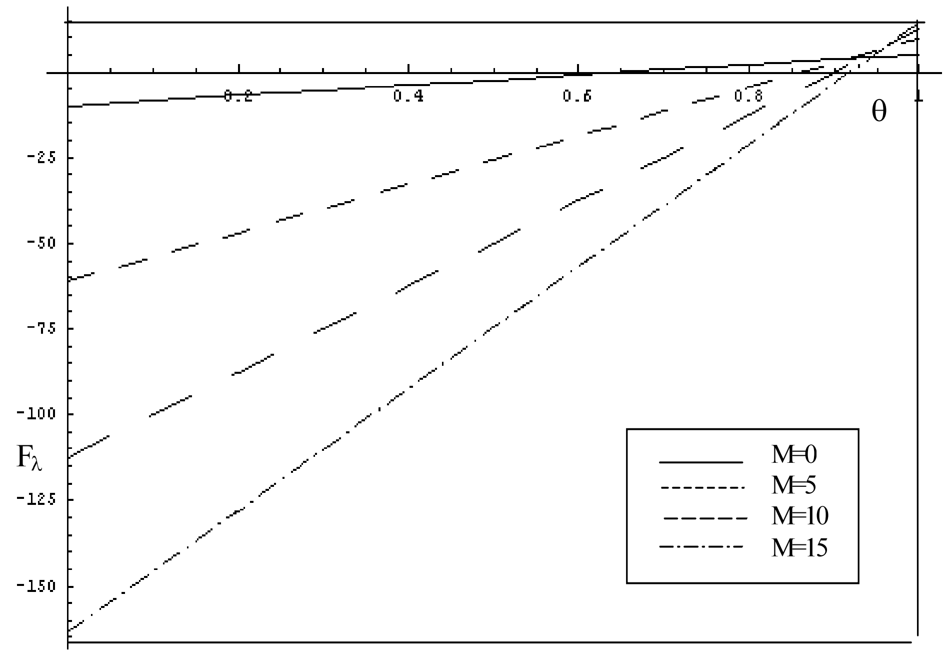

Fig.(4) Friction force versus flow rate at Re=10, k=1, φ=0.08 and δ=0.02.