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TM NONLINEAR ELECTROMAGNETIC WAVES IN SEMICONDUCTOR SUPERLATTICES WAVEGUIDING SYSTEMS

A.B. Abo-Shabab, and Mohammed M. Shabat and Samir S. Yassin
Department of physics, The Islamic University, Gaza, P.O.Box 108, Gaza Strip, Palestinian Authority, E-mail:Shabat@ mail.iugaza.edu

ملخـص: لقـد زاد الاهـتمام نحو دراسة الأمواج الكهرومغناطيسية الغير خطية التي تنتشر خلال طبقات الألياف الضوئية، وذلك لتطبيقاتها الكثيرة في الأجهزة التي تعمل بالأمواج القصيرة جداً.

فـي هذا السياق استخدمت مصفوفة الإنتقال لحاسب معادلة التشتت لتلك الأمواج التي تنتشر خلال أنصاف الموصلات المكونة من عدة طبقات ذات كثافة كهربية ودرجة توصيل مختلفة. وهذه الطبقات محاطة بغطاء من طبقة مغناطيسية غير خطية تعتمد فيها النفاذيه المغناذيه المغناطيسية داله في قيمه المجال المغناطيسي.

لقـد تـم حسـاب القيم العددية لعدد معين من الطبقات والتي تشكل نظام البلور التي يحتوي على عدد من الطبقات وهي تحدث زيادة مقابلة في كفاءة الأمواج السطحية والحجمية.

ونـأمل أن هـذا العمـل سـوف يساعد على تصميم العديد من الأجهزة الضوئية ذات المزايا والأغراض المختلفة.

Abstract: Considerable attention has been devoted to a nonlinear waves propagation in various wave-guide structure due to their applications in photonic-microwave devices $[1-5]$. In this communication, the dispersion characteristics of transverse magnetic polarized TM waves nonlinear guided waves propagating in a multilayer semiconductor superlattices waveguide surrounded by one side by a nonlinear magnetic cover have been investigated theoretically. The two sublattice uniaxial antiferromagnetic crystal is considered as a nonlinear magnetic medium where the permeability is treated as a function of the magnetic field. Nummerical results are demonstrated for a waveguiding system containing some number of layers of superlattices. We hope that our present work could lead to several extensions and promising applications in future technology.

I- Introduction

Recently a number of papers have appeared dealing with the propagation of bulk and surface plasmons in semiconductor superlattices of various types. Binary superlattices consisting of alternating layers of materials A and B with or without a two-dimensional electron (hole) gas at the interfaces where studied by many authors. A particular superlattice structure, the so-called n,i,p,i superlattices, has also been investigated and its particular features discussed.

In this paper we present a full theory of the bulk and surface-plasmon excitation spectrum of a finite superlattice covered by a nonlinear magnetic cladding. We have included the effects of both retardation and an external magnetic field, and we have obtained the dispersion relation for surface magnetoplasmon polaritons in this structure.

Our model is based on a transfer-matrix treatment, already presented earlier, to simplify the algebra, which is otherwise quite involved. Since the quantization of the electronic states into subbands is quite negligible due to our basic assumption that the layer thicknesses are sufficiently large, we can describe the properties of the layers by macroscopic dielectric functions. Thus the electromagnetic fields in each layer are described by solving Maxwell's equations subject to the appropriate boundary conditions.

The plan of this paper is as follows. Section II presents the full theoretical derivation of the magnetoplasmon polariton dispersion relation, assuming that the electromagnetic mode is $p(TM)$ polarized. Section III gives the power of the system in a special case. Section IV is devoted to the presentation of some numerical results.

II- General theory

The guiding structure to be considered consists of a nonlinear magnetic cladding in contact with superlattices everywhere on the $z=0$ plane, where the z axis points into the structure, the applied magnetic field is along y axis and the propagation is along x axis. The nonlinear magnetic cladding is assumed to be isotropic with a permeability given by:[4]

$$\mu_{NL} = \mu_L + \alpha|H|^2 = \mu_L + \alpha H_y^2 \tag{1}$$

where μ_L is the linear part of the permeability and α is the nonlinear coefficient. This expression arises from an expansion of the permeability about an applied static field H_0, and terms that could lead to harmonic generation are neglected. Hence, H is the ac magnetic field carried by the TM wave. H_y is real because only stationary, non-radiating waves will be considered.

We have a solution to the Maxwell's equations corresponding to a TM electromagnetic wave propagating in the nonlinear cladding: [4]

$$H_y(z) = \frac{1}{k_0}\sqrt{\frac{2}{\alpha\varepsilon_s}} \cdot \frac{\alpha_s}{\cosh[k_s(z-z_0)]} \tag{2}$$

where z_0 is a constant of integration that defines the position of a self-focused peak in H_y and $\alpha_s = \sqrt{k_x^2 - k_0^2\varepsilon_s\mu_L}$, ε_s is the dielectric constant of the nonlinear medium .

The semiconductor superlattice that is considered in this paper are multilayer materials in cells along z direction. Materials a and c are n type and p

type, with dielectric constants $\varepsilon_a(\omega)$ and $\varepsilon_c\ (\omega)$,and with thickness a and c respectively. Materials b and d are intrinsic semiconductors with frequency independent tensor ε_b and ε_d and thickness b and d respectively. The unit cell has length $L = a + b + c + d$ and is designated by the index n .

In the n' th unit cell, at the interfaces $z = nL$ and $z = nL + a$ there is a two-dimensional electron gas, while at $z = nL + a + b$ and $z = nL + a + b + c$ there is a two dimensional hole gas. We assume that a uniform external magnetic field is imposed in the y direction and that surface magnetoplasmon polaritons are allowed to propagate in the x direction with a wave-vector k and frequency ω .

We are going firstly to find the dispersion relation for the system in an infinite superlattice and then for a finite one. In both cases the field amplitudes are assumed to be localized at each interface. In the following we discuss bulk modes and surface modes.

1-Bulk modes

In this section, we present the general theory for calculating the bulk and surface plasmon dispersion relations for a superlattices in the presence of a static magnetic field applied parallel to the material interfaces. The x component of the electric field and the y component of the magnetic field in each layer of the n' th cell is given by:

$$E^n_{xj}\left(z\middle|k\omega\right) = A^n_{1j}e^{-\alpha_j z} + A^n_{2j}e^{\alpha_j z}\ , \tag{3}$$

$$H^n_{yj}\left(z\middle|k\omega\right) = -i\frac{\omega\varepsilon_0\varepsilon_j}{\alpha_j}\left[A^n_{1i}e^{-\alpha_j z} - A^n_{2j}e^{\alpha_j z}\right] \tag{4}$$

where

$$\varepsilon_j = \varepsilon_j(\omega) =$$

$$\begin{bmatrix} \varepsilon_1 & 0 & i\varepsilon_2 \\ 0 & \varepsilon_3 & 0 \\ -i\varepsilon_2 & 0 & \varepsilon_1 \end{bmatrix}, \varepsilon_1 = \varepsilon_{\infty j}\left[1 + \frac{\omega_p^2}{\omega_c^2 - \omega^2}\right], \varepsilon_2 = \varepsilon_{\infty j}\left[\frac{\omega_c\omega_p^2}{\omega(\omega_c^2 - \omega^2)}\right], \varepsilon_3 = \varepsilon_{\infty j}\left[1 - \frac{\omega_p^2}{\omega^2}\right],$$

$$j = a, b, c, \text{or } d \ \text{ and } \ \alpha_j = \begin{cases} \left(k_x^2 - \varepsilon_{vj}\omega^2/c^2\right)^{1/2}, k_x > \varepsilon_j\omega/c \\ i\left(\varepsilon_{vj}\omega^2/c^2 - k_x^2\right)^{1/2}, k_x < \varepsilon_j\omega/c \end{cases} \quad , \quad \varepsilon_v = \varepsilon_{xx} + \frac{\varepsilon_{xz}^2}{\varepsilon_{xx}}$$

The boundary conditions for the electromagnetic interfaces:

$$z = nL + a, nL + a + b, nL + a + b + c \quad and \quad z = (n+1)L$$

give the following equations [6]

$$A_{1a}^{n} f_a + A_{2a}^{n} \bar{f}_a = A_{1b}^{n} + A_{2b}^{n}, \tag{5}$$

$$\varepsilon_a'\left(A_{1a}^{n} f_a - A_{2a}^{n} \bar{f}_a\right) = \left(\varepsilon_b' + \sigma_h\right) A_{1b}^{n} - \left(\varepsilon_b' + \sigma_h\right) A_{2b}^{n}, \tag{6}$$

$$A_{1b}^{n} f_b + A_{2b}^{n} \bar{f}_b = A_{1c}^{n} + A_{2c}^{n}, \tag{7}$$

$$\varepsilon_b'(A_{1b}^{n} f_b - A_{2b}^{n} \bar{f}_b) = (\varepsilon_c' - \sigma_e) A_{1c}^{n} - (\varepsilon_c' + \sigma_e) A_{1c}^{n} - (\varepsilon_c' + \sigma_e) A_{2c}^{n}, \tag{8}$$

$$A_{1c}^{n} f_c + A_{2c}^{n} \bar{f}_c = A_{1d}^{n} + A_{2d}^{n}, \tag{9}$$

$$\varepsilon_c'(A_{1c}^{n} f_c - A_{2c}^{n} \bar{f}_c) = (\varepsilon_d' - \sigma_e) A_{1d}^{n} = (\varepsilon_d' + \sigma_e) A_{2d}^{n}, \tag{10}$$

$$A_{1d}^{n} f_d + A_{2d}^{n} \bar{f}_d = A_{1a}^{n+1} + A_{2a}^{n+1}, \tag{11}$$

$$\varepsilon_d'(A_{1d}^{n} f_d - A_{2d}^{n} \bar{f}_d) = (\varepsilon_a' - \sigma_h) A_{1a}^{n+1} - (\varepsilon_a' + \sigma_h) A_{2a}^{n+1} \tag{12}$$

where

$$\varepsilon_j' = \frac{\varepsilon_j}{\alpha_j},\ \sigma_p = \frac{n_p e^2}{m_p^* \omega^2 \varepsilon_0},\ p = e, h. \quad f_j = e^{-\alpha_j j}, \quad \bar{f}_j = e^{\alpha_j j}$$

and m_p^* is the effective mass for the electron and hole.
For each medium, the column vector can be written as

$$\left| A_j^n \right\rangle = \begin{bmatrix} A_{1j}^n \\ A_{2j}^n \end{bmatrix}, \tag{13}$$

The equations (5-12) can be written in a matrix form as

$$\begin{aligned} M_a \left| A_a^n \right\rangle &= N_b \left| A_b^n \right\rangle, \\ M_b \left| A_b^n \right\rangle &= N_c \left| A_c^n \right\rangle, \\ M_c \left| A_c^n \right\rangle &= N_d \left| A_d^n \right\rangle, \\ M_d \left| A_d^n \right\rangle &= N_a \left| A_a^{n+1} \right\rangle, \end{aligned} \tag{14}$$

where we have defined the matrices

$$M_j = \begin{bmatrix} f_j & \bar{f}_j \\ \varepsilon'_j f_j & -\varepsilon'_j \bar{f}_j \end{bmatrix}, \tag{15}$$

and

$$N_j = \begin{bmatrix} 1 & 1 \\ \varepsilon'_j - \sigma_p & -\varepsilon'_j - \sigma_p \end{bmatrix}, \tag{16}$$

From the equations (14-16) it is easy to see that

$$\left| A_j^{n+1} \right\rangle = T \left| A_j^n \right\rangle, \tag{17}$$

where the matrix T is given by

$$T = N_a^{-1} M_d N_d^{-1} M_c N_c^{-1} M_b N_b^{-1} M_a \tag{18}$$

The matrix T in the equation (18) is a transfer matrix because it relates the coefficients of the electric field in one cell to those in the preceding cell. Taking into account the translational symmetry of the problem, we can use Bloch's theorem, that is

$$\left| A_j^{n+1} \right\rangle = e^{iQL} \left| A_j^n \right\rangle. \tag{19}$$

By using the equations (17) and (18) we have

$$T \left| A_j^n \right\rangle = e^{iQL} \left| A_j^n \right\rangle, \tag{20}$$

$$T^{-1} \left| A_j^n \right\rangle = e^{-iQL} \left| A_j^n \right\rangle, \tag{21}$$

and consequently

$$\left[\cos(QL) I - \frac{1}{2}\left(T + T^{-1}\right) \right] \left| A_j^n \right\rangle = 0 \tag{22}$$

Since $\left| A_j^n \right\rangle$ is a general vector of the structure considered, the dispersion relation of the bulk polaritons on the superlattice will be given by

$$\cos(QL)I = \frac{1}{2}(T + T^{-1}) \tag{23}$$

From the definition of the transfer matrix in equation (18) and using the equations (15) and (16) we can show that $\det(T)=1$, therefore

$$T^{-1} = \begin{bmatrix} T_{22} & -T_{12} \\ -T_{21} & T_{11} \end{bmatrix} \tag{24}$$

and hence, from the equations (23) and (24) our dispersion relation for the bulk modes is simply

$$\cos(QL) = \frac{1}{2}Tr(T) \tag{25}$$

where Q is the Bloch wave vector governing the phase of the wave from one unit cell to the next ,and L is the length of the unit cell and T is the transfer matrix which will give the potentials in the $n+1$ cell in terms of the n cell as the following :

$$\begin{bmatrix} A_{1j}^{n+1} \\ \\ A_{2j}^{n+1} \end{bmatrix} = T \begin{bmatrix} A_{1j}^{n} \\ \\ A_{2j}^{n} \end{bmatrix} \tag{26}$$

2- Surface modes

In order to study the surface modes we match the boundary conditions for the electric and magnetic fields at the surface where $z = 0$. Then the periodicity in the z direction is destroyed and we can no longer assume Bloch's theorem , therefore we have to consider electromagnetic modes having their excitations localized in the near vicinity of the interface between nonlinear magnetic cladding and superlattices. We replace Q by $i\beta$ then eq.(25) becomes

$$\cosh(\beta L) = \frac{1}{2}TrT$$

We can conclude this result in the vector form as:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{bmatrix} A_{1a}^{0} \\ A_{2a}^{0} \end{bmatrix} = e^{-\beta L} \begin{bmatrix} A_{1a}^{0} \\ A_{2a}^{0} \end{bmatrix} \tag{27}$$

$$T_{11}A^0_{1A} + T_{12}A^0_{2a} = e^{-\beta L}A^0_{1a}$$
$$T_{21}A^0_{1a} + T_{22}A^0_{2a} = e^{-\beta L}A^0_{2a} \qquad (28)$$

if we replace $\frac{A^0_{1a}}{A^0_{2a}}$ by λ we have the dispersion equation of the surface modes,

$$\lambda\{T_{11} + \lambda T_{12}\} = T_{21} + \lambda T_{22} \qquad (29)$$

where:

$$\lambda = \frac{\alpha_s \tanh(\alpha_s z_0) - \alpha_a / \varepsilon_a}{\alpha_s \tanh(\alpha_s z_0) + \alpha_a / \varepsilon_a},\ \alpha_s = \sqrt{k_x^2 - k_0^2 \varepsilon_s \mu_L}$$

III- Special case

If we consider $a = c$, $b = d$, $\sigma_h = \sigma_e = 0$,and $\varepsilon_a(\omega)$ is frequency dependent but ε_b is frequency independent, therefore the periodicity of the superlattice is $L = a + b$ And the dispersion equation of the surface in this case is

$$\cos(Q(a+b)) = \frac{1}{2}Trace(T_1) \qquad (30)$$

and in this case the power of the system is:

1-the power of the nonlinear medium is

$$p_{NL} = \frac{k_x \alpha_s}{\alpha\omega k_0^2 \varepsilon_0 \varepsilon_s^2} \qquad (31)$$

2- the power of superlattices is the sum of all powers in each layer of superlattices as

$$p_{\sup erlattices} = \sum_{n=0}^{N}\left(A^n_{1a}\right)^2\left[\frac{1}{2\alpha_a}\left(1 - e^{-2\alpha_a a}\right)\right] + \sum_{n=0}^{N}\left(A^n_{2a}\right)^2\left[\frac{1}{2\alpha_a}\left(e^{2\alpha_a a} - 1\right)\right] -$$

$$2\sum_{n=0}^{N} A^n_{1a}.A^n_{2a}.a +$$

$$\sum_{n=0}^{N}\left(A^n_{1b}\right)^2\left[\frac{1}{2\alpha_b}\left(1 - e^{-2\alpha_b b}\right)\right] + \sum_{n=0}^{N}\left(A^n_{2b}\right)^2\left[\frac{1}{2\alpha_b}\left(e^{2\alpha_b b} - 1\right)\right] - 2\sum_{n=0}^{N} A^n_{1b}.A^n_{2b} b \qquad (32)$$

where $A^{n}_{1,2,a,b}$ are constants can be found from the boundary conditions using the surface constants in eq.(26) and the surface constants are:

$$A^0_{1a} = \frac{i(\alpha_s)}{2}\sqrt{\frac{2}{\alpha\varepsilon_s}\frac{1}{\omega\varepsilon_0 k_0}\frac{1}{\cosh(\alpha_s z_0)}}\left[\alpha_s \tanh(\alpha_s z_0) + \frac{\alpha_a}{\varepsilon_a}\right]$$

$$A^0_{2a} = \frac{i(\alpha_s)}{2}\sqrt{\frac{2}{\alpha\varepsilon_s}\frac{1}{\omega\varepsilon_0 k_0}\frac{1}{\cosh(\alpha_s z_0)}}\left[\alpha_s \tanh(\alpha_s z_0) - \frac{\alpha_a}{\varepsilon_a}\right]$$

$$P_{total} = P_{nonlinear} + P_{\sup erlattices} \tag{33}$$

IV-Numerical results

In this section we present numerical examples of dispersion relations of magnetoplasmons in superlattices. We will show that the effect of quasiperiodic layering is to increase the number of bulk bands and surface modes. And show that the new surface modes are nonreciprocal with respect to propagation direction in the presence of an applied magnetic field.

In order to obtain numerical results we consider the dielectric materials a and c as Si doped with n and p impurities. Since we do not use highly semiconductors, we assume that $\varepsilon_a(\omega) = \varepsilon_c(\omega)$, and the dielectric constant of the Si can be taken as $\varepsilon(\omega) = \varepsilon_L(1 - \omega^2/\omega_p^2)$. Where $\varepsilon_L = 11.7$ is the background dielectric constant of the material, and $\omega_p = 7.65 \times 10^{13} s^{-1}$ is the electronic plasma frequency and we consider $\varepsilon(\omega)$ independent of the impurity density. The effective mass of the electrons and holes are related to the electron mass m_0 by $m_e^* = 0.2m_0, m_h^* = 0.4m_0$ respectively. We also assume that the dielectrics b and d consists of SiO_2 with dielectric constant $\varepsilon_b = \varepsilon_d = 3.7$.

Fig.(1) shows the dispersion for the surface modes with an applied field given by $\omega_C = 4.075 meV$. Both $\pm k$ are shown, and there are several points of interest.

Fig.(2) The frequency (ω/ω_p) of the two lower and upper bands of the bulk polaritons as a function of $k_x a$, for superlattice. we plot the dispersion relation for surface modes and bulk polaritons by considering the semiconductor layers (n and p) with 400 A^0 of thickness and the insulators with 200 of thickness and with: $|\sigma_e| = |\sigma_n| = 2\times10^{16}\ carrier/m^2$,

$\mu_L = 1.29, \alpha = 8.869 \times 10^{-8} m^2 A^{-2}\ c^{-2}$. We observed that the existence of four bands tend to crowed together when k increases.

Fig.(3) illustrates the dependence of αH_y^2 on the dimensionless coordinate $k_0 z$ for two values of the propagation constant β we see that for $\beta = 3.90$,the maximum point of the curve becomes greater than the other curve when $\beta = 3.5$.

Fig.(4) the power flow versus wave index for TM surface guided waves at the interface between a nonlinear magnetic cladding and the first unit cell in superlattices . Note that we have three curves according to three different values of α such that the decrease value of α the upper curve becomes.

V-Conclusion

In the present work we have discussed the nonlinear waves, propagating in semiconductor superlattices covered by a nonlinear cladding. A transfer matrix is used to simplify the algebraic equations. We then derived the dispersion equation of the surface and the bulk modes. The power of the system in a special case was calculated. Numerical results have shown the effect of the quasiperiodic layering is to increase the number of bulk bands and surface modes. A dispersion curves for surface and bulk modes is displayed.

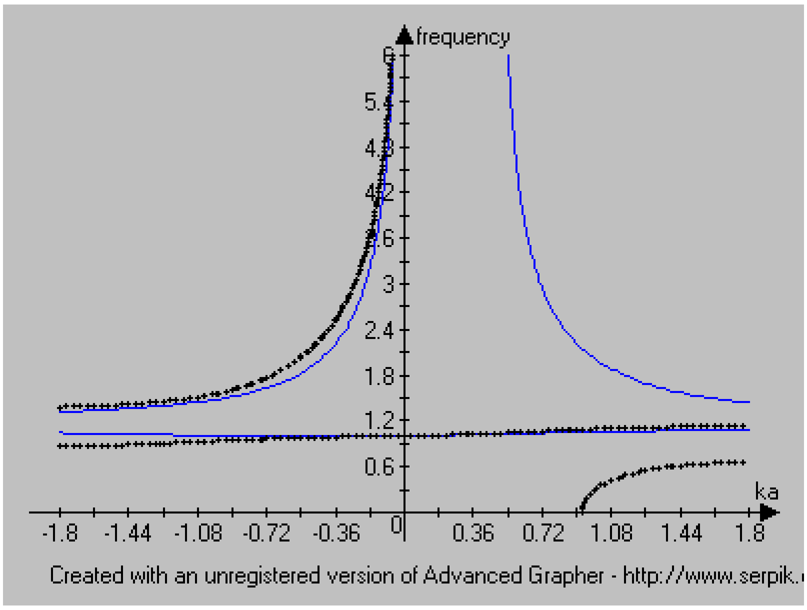

Fig.(1): computed the dispersion curves for surface modes.

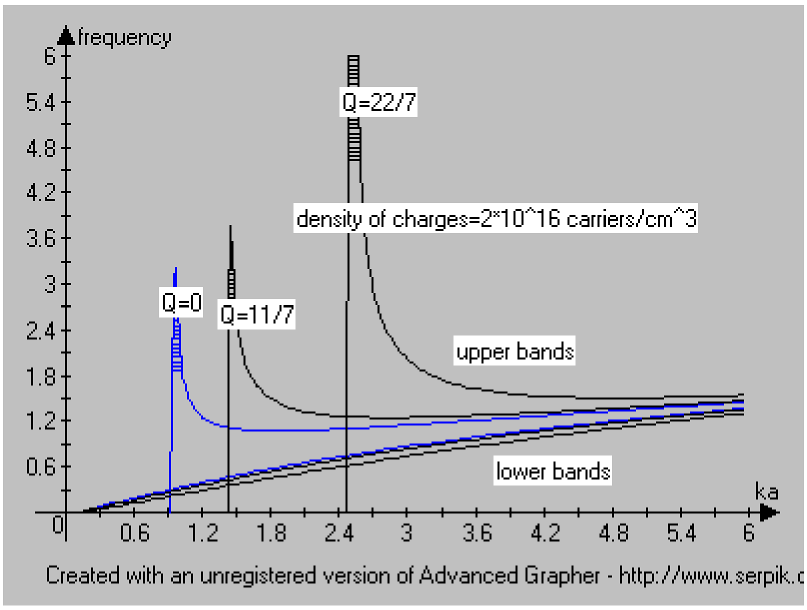

Fig.(2): computed the dispersion curves for bulk modes

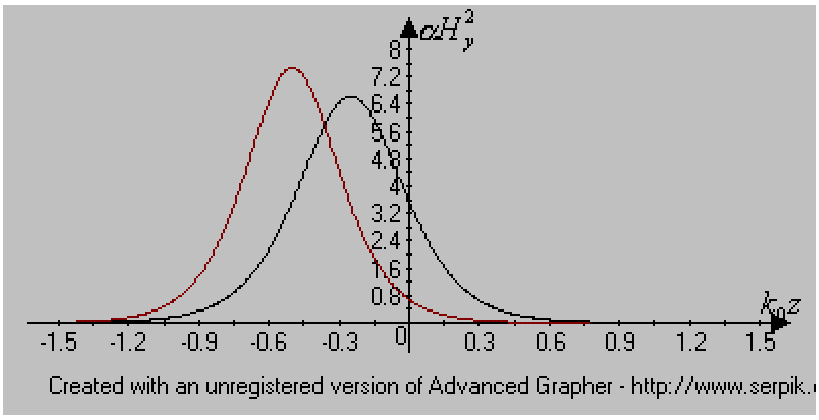

Fig.(3): The nonlinearity interface αH_y^2 versus $k_0 z$

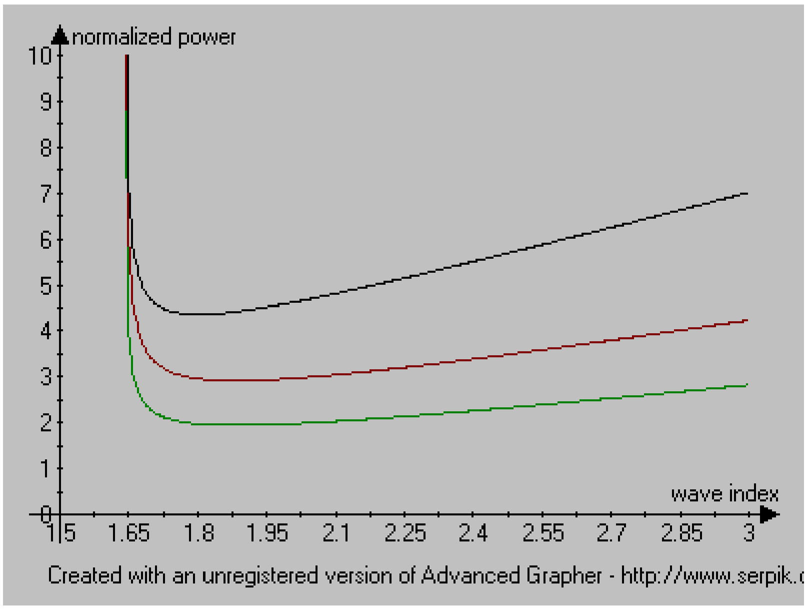

Fig.(4): The normalized power flow versus wave index $\beta = k_x / k_0$

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