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CONSTANT-WEIGHT CODES USING METASYMPLECTIC SPACE $F_{4,1}(q)$ AND ITS RESIDUE

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ملخـص : نولـد في هذا البحث بعض مجموعات الترميز غير الخطية الثنائية ذات الوزن الثابت و ذلك باسـتخدام الهندسـة الميتاسـمبلكتية ذات الخطوط آلتي عليها نقطتين و ذات الخطوط آلتي عليها اكثر من نقطتين و كذلك استخدمنا الباقي عند نقطة لها.

Abstract: In this paper we generate few families of non-linear binary constant-weight codes using the metasymplectic spaces $F_{4,1}(q)$ and its residue.

Introduction

The beginning of coding theory goes back to the middle of this century with the work of Golay, Hamming and Shannon. Although it has its origins in engineering and applied problems, the subject has developed by using more mathematical techniques.

In recent year there has been an increasing interest in finite spaces and important applications to practical topics such as coding theory.

Many papers have taken the algebraic concepts for purpose of obtaining codes, here we have used the geometric means to construct some families of binary non-linear constant-weight codes, at the same time there are many geometries that can be used for constructing such kinds of codes.

In this paper we construct non-linear binary constant-code arising from the residue of metasymplectic space.

Basic geometry Definitions

A point-line geometry $\Gamma = (P, L)$ is a pair of sets, P is called the set of “points” and L is called the set of “lines”, where members of L are just subsets of P. If p is a point belongs to a line l we say that p lies on l or l passes through p or p is incident with l. If p, q are two points on one line l we say that p and q are collinear and this is denoted by $p \sim q$. $\Gamma = (P, L)$ is called linear (singular) space if each pair of distinct points lie exactly on one line. Γ is called partial (or near) linear if each pair of points lie on at most one line. A subspace of a point-line geometry $\Gamma = (P, L)$ is a subset X of points together with all lines l in L such that if l has at least two points of X then l lies entirely in X. A path of length k from x_0 to x_k is a set of k +1 points $x_0, x_1, x_2, ..., x_k$, such that x_i is collinear with x_{i+1}, i = 1, 2, 3, ..., k-1.

A geodesic is a shortest path between two points. We define the distance function d: $p \times p \to Z$ by d (x, y) = the length of any geodesic from x to y. A subspace X is called convex if it contains all geodesics between any two points of X. The smallest subspace containing a set X is called the subspace generated by X and is denoted by $\langle X\rangle$. If p is a point, $p^{\perp}$ means all point collinear with p in addition to p itself. $\Delta_k(p) = \{x \in p \mid x$ is at distance k from p$\}$. $\Delta^*_k(p) = \{x \in P \mid x$ is of distance at most k from p$\}$. Let Γ be point-line geometry. A geometric hyperplane of Γ is a proper subspace with the property that every line of Γ meets it in at least one point. A hyperplane of Γ is a maximal proper subspace of Γ.

Some basic space

$\Gamma = (P, L)$ is called a gamma space if $x^{\perp}$ is a subspace for every point $x \in P$ **.**
A polar space is a point-line geometry that satisfies the following Buekenhout- Shult axiom:
(B-S) for each point p not incident with a line *l* ; p is collinear with one or all points of *l*.
If $\Gamma = (P, L)$ is a point-line geometry; $Rad(\Gamma) = \{ q \in p \mid$ p collinear to q for all $p \in p \}$. Rank of Γ is the largest integer n for which there is a chain of singular subspaces $\{X_i\}$, i = 1, 2, ...n, such that:
$X_1 \subset X_2 \subset ... \subset X_n$, where $X_i \neq X_j$, $i \neq j$, and if there is no such integer; the rank of Γ is infinite. If Γ is a polar space and $Rad(\Gamma) = \varnothing$, then Γ is called non-degenerate polar space; otherwise Γ is called degenerate polar space. A point-line geometry is called a parapolar space of rank r + 1, $r \geq 2$; if it satisfies the following conditions:

(pp1) Γ is a connected gamma space.

(pp2) for every line l; $l^{\perp}$ is not a singular space.

(pp3) for every pair of distinct points x, y; $x^{\perp} \cap y^{\perp}$ is either empty, a point, or a non-degenerate polar space of rank r.

A strong parapolar space is a parapolar space in which $x^{\perp} \cap y^{\perp}$ is a polar space for every pair of points distinct x, y of distance 2 apart.
If x, y are two points of a parapolar space; (x, y) is called a special pair if $x^{\perp} \cap y^{\perp}$ is just one point, and (x, y) is called a polar pair if $x^{\perp} \cap y^{\perp}$ is a non-degenerate polar space of rank at least 2.
Let p be a point in a point-line geometry $\Gamma = (P, L)$; Residue of Γ at p denoted by Γ_p or Res(p); is a point-line geometry (P_p, L_p) defined as follows: P_p is the set of all lines containing p; a member of L_p is the set of all lines containing p and contained in a plane (singular space of rank 3).

DEFINITION: (Metasymplectic space). A Metasymplectic space is a set P in which some subsets called lines, planes, and symplecta are distinguished, and satisfy the following axioms:

(M1) the intersection of distinct symplecta is empty, a point, a line, or a plane.

(M2) A symplecton S together with its "singular spaces"; points, lines, and planes contained in S is a polar space of rank 3.

(M3) Considering the set x^* of all symplecta containing a given point $x \in P$, and calling lines (resp. Planes) of x^* the subset of x^* consisting of all symplecta of x^* containing a plane (resp. a line) through x, we also obtain a polar space of rank 3

Basic Algebraic Definitions and Notations

Let B be a symmetric or alternate bilinear form defined on a vector space V over an arbitrary field F. For (a subspace) $W \subset V$; we set

$W^{\perp} = \{u \in V : B(u, v) = 0, \text{ for all vectors } v \in V\}$. $V^{\perp}$ is called radical of V with respect to B. A bilinear form B on a vector space V is called non-degenerate iff $V^{\perp} = \{0\}$. Otherwise B is called degenerate.

Two forms B_1, B_2 on V are said to be equivalent if there is a one-to-one and onto linear transformation $\psi : V \rightarrow V$ such that:$B_1(u, v) = B_2(\psi(u), \psi(v))$.

A vector $u \in V$ is called an isotropic vector, if $B(u, u) = 0$, and a subspace W of V is called totally isotropic (abbreviated TI) subspace of V if $B(u, v) = 0$ for all $u, v \in W$. If a TI subspace W of V is not contained properly in any TI subspace of V; W is called maximal totally isotropic (abbreviated MTI) subspace of V.

It can be shown that all the MTI subspaces have the same dimension it is called witt index of V and is denoted by ind(V). Two vectors u, v are called orthogonal if $B(u, v) = 0$.

A 2-dimensional vector space with non-degenerate bilinear form B, in which there is an isotropic vector u is called a hyperbolic plane, otherwise it is called an anisotropic plane.

A vector space V of dimension 2n is called hyperbolic if V is endowed with a symmetric bilinear form of witt index n, and is called elliptic if witt index is $n - 1$.

The following couple Theorems explains the structure of vector spaces endowed with bilinear forms.

THEOREM. Let B be a non-degenerate symmetric bilinear form on a vector space V of dimension 2n over a finite field F. Then B is a hyperbolic form on V iff V has a basis A, such that $V = H_1 \perp H_2 \perp \ldots \perp H_n$, where all H_i are hyperbolic planes, $i = 1, 2, \ldots, n$ with $ind(V) = n$.

It shows that all hyperbolic non-degenerate symmetric bilinear forms on a certain vector space are equivalent. To find one we will revert to following theorem that determines when the scalar product is hyperbolic.

THEOREM. Let B be the Euclidean scalar product on a vector space of dimension $n = 2r$ over the finite field F of order k. Then if both k and r are odd

(i) B is a hyperbolic form iff $k \equiv 1 \pmod 4$.

(ii) B is an elliptic form iff $k \equiv 3 \pmod 4$.

Furthermore, if r is even integer, or q is even; B is always a hyperbolic form.

Dual polar spaces

From the definition of the metasymplectic space the residue of metasymplectic space at any point is dual polar space (C_3)

The dual polar space is the space whose point are maximal singular spaces of classical polar space of rank at least two, lines are all totally singular subspaces of dimension one less than the dimension of a maximal singular space.

All symplecta of these geometries are generalized quadrangles (Quad)

LEMMA:[CC] Let $\Gamma = (P, L)$ be a dual polar space, of rank 3 the following holds.

i) Γ is a gamma space whose lines are maximal cliques
ii) (P) Holds.
iii) Each pair of points at mutual distance 2 is contained in a unique quad.
iv) Each pair of quads has either empty intersection or meets in a line.
v) For any point $x \notin Q \Rightarrow x^{\perp} \cap Q = \varnothing$ or one point.
vi) The diameter of Γ is 3.

Note: This lemma is true for the dual polar space that comes from thick polar space of rank 3.

CLASSICAL EXAMPLES OF FINITE POLAR SPACES:

Let V be a vector space over a finite field $F = GF(q)$, q is a prime power.

SYMPLECTIC GEOMETRY $W_n(q)$ is the point-line geometry (P, L), where P is the set of all one dimensional subspaces $\langle x \rangle$ of V, and L is the set of all two dimensional subspaces $\langle x, y \rangle$ for which $B(x, y) = 0$, for a symplectic bilinear form B. In this case n is even, the polar space is of rank n/2.

HYPERBOLIC GEOMETRY $\Omega^{+}_{n}(q)$ is the point-line geometry (P, L), where P is the set of all one dimensional subspaces $\langle x \rangle$ of V for which $B(x, x) = 0$, and L is the set of all two dimensional subspaces

$\langle x, y\rangle$ for which $B(x, y) = 0$, for a hyperbolic bilinear form B. In this case n is even, the polar space is of rank n/2.

ELLIPTIC GEOMETRY $\Omega^{-}_{n}(q)$ is the point-line geometry (P, L), where P is the set of all one dimensional subspaces $\langle x\rangle$ of V for which $B(x, x) = 0$, and L is the set of all two dimensional subspaces $\langle x, y\rangle$ for which $B(x, y) = 0$, for a elliptic bilinear form B. In this case n is even, the polar space is of rank (n/2) - 1.

ORTHOGONAL GEOMETRY $\Omega_n(q)$ is the point-line geometry (P, L), where P is the set of all one dimensional subspaces $\langle x\rangle$ of V for which $B(x, x) = 0$, and L is the set of all two dimensional subspaces $\langle x, y\rangle$ for which $B(x, y) = 0$, for an orthogonal bilinear form B. In this case n is odd, the polar space is of rank (n-1)/2.

HERMITIAN GEOMETRY $H^{+}_{n}(q^2)$ is the point-line geometry (P, L), where P is the set of all one dimensional subspaces $\langle x\rangle$ of V for which $B(x, x) = 0$, and L is the set of all two dimensional subspaces $\langle x, y\rangle$ for which $B(x, y) = 0$, for a Hermitian bilinear form B. In this case n is even, the polar space is of rank n/2.

HERMITIAN GEOMETRY $H^{-}_{n}(q^2)$ is the point-line geometry (P, L), where P is the set of all one dimensional subspaces $\langle x\rangle$ of V for which $B(x, x) = 0$, and L is the set of all two dimensional subspaces $\langle x, y\rangle$ for which $B(x, y) = 0$, for a Hermitian bilinear form B. In this case n is odd, the polar space is of rank (n-1)/2.

THEOREM : [Th] The number of points of the finite classical polar spaces are given by the following formulae:

$|W_{2n}(q)| = (q^{2n}- 1) / (q – 1)$,

$|\Omega(2n + 1, q)| = (q^{2n} – 1) / (q – 1)$,

$|\Omega^{+}(2n, q)| = (q^{2n} + 1) (q^{n} – 1) / (q – 1)$,

$|\Omega^{-}(2n, q)| = (q^{n-1} - 1) (q^{n} + 1) / (q – 1)$,

$|H^{-}(2n +1, q^2)| = (q^{2n+1} + 1) (q^{2n+1} - 1) / (q^2 – 1)$,

$|H^{+}(2n, q^2)| = (q^{2n} - 1) (q^{n} + 1) / (q^2 – 1)$,

THEOREM: [Th] The numbers of maximal totally isotropic subspaces or maximal singular subspaces of the finite classical polar spaces are given by the following:

$|W_{2n}(q)| = (q + 1)(q^2 + 1)\ldots\ldots\ldots(q^{(n+1)/2} +1)$,

$|\Omega(2n + 1, q)| = (q +1)(q^2 +1)\ldots\ldots(q^{n} +1)$

$|\Omega^{+}(2n, q)| = 2(q + 1) (q^2 – 1) \ldots\ldots.(q^{n} + 1)$,

$|\Omega^{-}(2n, q)| = (q^2 + 1) (q^3 + 1) \ldots\ldots.(q^{2n+1} + 1)$,

$|H^{-}(2n +1, q^2)| = (q^3 + 1) (q^5 + 1) \ldots\ldots (q^{2n+1} + 1)$,

$|H^+(2n, q^2)| = (q + 1)(q^3 + 1) \ldots\ldots (q^{2n+1} + 1)$,

THEOREM: [Th] Let V be a vector space equipped with a bilinear form then the number of totally isotropic k-subspaces is the following:

1- $\begin{bmatrix} n \\ k \end{bmatrix}_q \prod_{i=0}^{k-1} \left(q^{n-i} + 1\right)$ in the symplectic case W(2n, q).

2- $\begin{bmatrix} n \\ k \end{bmatrix}_q \prod_{i=0}^{k-1} \left(q^{n-i} + 1\right)$ in the orthogonal case $\Omega(2n + 1, q)$.

3- $\begin{bmatrix} n \\ k \end{bmatrix}_q \prod_{i=0}^{k-1} \left(q^{n-i-1} + 1\right)$ in the hyperbolic case $\Omega^+(2n, q)$.

4- $\begin{bmatrix} n \\ k \end{bmatrix}_q \prod_{i=0}^{k-1} \left(q^{n-i+1} + 1\right)$ in the elliptic case $\Omega^-(2n+2, q)$, where

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{\prod_{i=0}^{k} \left(q^{n-1} - 1\right)}{\prod_{i=1}^{k} \left(q^{i} - 1\right)}.$$

Now the table below lists the number of point, quad and the numbers of points in each quad in dual polar space of rank 3.

Type	# of points	# of quad	# of points in each quads
$W_6(q)$	$(q+1)(q^2+1)(q^3+1)$	$(q^6 - 1) / (q - 1)$	$(q^4 - 1) / (q - 1)$
$\Omega(7, q)$	$(q+1)(q^2+1)(q^3+1)$	$(q^6 - 1) / (q - 1)$	$(q^4 - 1) / (q - 1)$
$\Omega^+(6, q)$	$2(q+1)(q^2+1)(q^3+1)$	$(q^2+1)(q^3-1) / (q - 1)$	$(q+1)(q^2-1) / (q - 1)$
$\Omega^-(6, q)$	$(q^2+1)(q^3+1)(q^4+1)$	$(q^2-1)(q^3+1) / (q - 1)$	$(q-1)(q^2+1) / (q - 1)$
H-(7, q2)	(q3+1)(q5+1)(q7+ 1)	(q7+1)(q7-1) / (q2– 1)	(q5+1)(q5-1) / (q2 – 1)
$H^+(6, q^2)$	$(q+1)(q^3+1)(q^5+1)$	$(q^6 - 1)(q^6+1) / (q^2 - 1)$	$(q^4 - 1)(q^4+1) / (q^2 - 1)$

TABLE 1

CONSTRUCTION OF THE CODES

In this section we shall construct the non- linear binary constant-weight codes that are arising from the metasymplectic space and its residue; the dual polar space $C_{3,3}(q)$ i.e. $\Omega(7, q)$ which comes from the orthogonal polar space. First we will introduce coding theory terminology.

DEFINITION: Let A = $\{a_1, a_2, a_3, \ldots\}$ be a finite set, called a code alphabet, and let A^n be the set of all strings of length n over A. Then any nonempty subset C of A^n is called a code. Members of C are called codewords. If $C \subset A^n$ contains M codewords, then C is said to have length n and size M and written (n, M)-code.

DEFINITION: Let x and y be two strings of the same length, over the same alphabet, the Hamming distance d(x, y) between x and y is the number of positions in which x and y differ. If d = minimum {d (x, y): x, y $\in$C, x $\neq$ y}; d is called the minimum distance of C, in this case we say that C is (n, M, d)-code.

We are concerned mainly on codes over finite fields, so, we can define the following:

DEFINITION: The weight wt(x) of a word x $\in$ C is the number of non-zero positions in x

DEFINITION: If C is a linear vector subspace of F^n; C is called a linear code and if the dimension of C is k we say that C is [n, k, d]-code. If all codewords in C have the same Hamming weight w, then C is called a constant-weight code and we say that C is [n, M, d, w]-code.

The incidence matrix is a matrix whose columns are labeled by points of a certain geometry and whose rows are labeled by certain sets of subspaces, the position corresponding to the intersection of the column labeled p and the row labeled S will be 1 if the point p is incident with the subspace S, and it will be 0 if the point p is not incident with the subspace S. In this paper we will use three incidence matrices; one for the dual polar space whose columns are labeled by the points of the dual polar space and whose rows are labeled by the quads in the dual polar space and will be called the **dual point-quad incidence matrix**, the second is the matrix whose columns are labeled by the points of the thin metasymplectic space and whose rows are labeled by the symplecta in the metasymplectic space and will be called the **thin point-symp incidence matrix**, the third is the same as the second for the thick metasymplectic space and will called the **thick point-symp incidence matrix.**

THEOREM

(a) *The rows of dual point-quads incidence matrix represent an (n, M, d, w)-non-linear binary constant-weight code of parameters:*

$n = (q+1)(q^2+1)(q^3+1),$

$M = (q^6 - 1)/(q-1),$

$d = 2(q^4 - 1)/(q-1) - 2(q+1),$

$w = (q^4 - 1)/(q-1).$

(b) *The columns of the of dual point-quads incidence matrix form a constant weight code of parameters:*

$n = (q^6 - 1)/(q-1)$

$M = (q+1)(q^2+1)(q^3+1)$

$d = 2(q^3 - 1)/(q-1) - 2\times(q+1)$

$w = (q^3 - 1) / (q – 1)$

Proof. (a) Let G be the dual point-quads incidence matrix. All quads have the same cardinality, it follows that rows of G have the same weight. The weight of each row is the number of points in the quad corresponding to the polar space $\Omega(7, q)$ thus by table 1, $w = (q^4 - 1) /(q – 1)$;.

Two rows of G have 1 in j^{th} column if the point p_j is incident with both quad that corresponds to both rows, this means that the point is in the intersection of both quad. By Lemma 4.1.1, two quads either intersect in a line or they are disjoint. It follows that the corresponding two rows differ in $|Q_1|+|Q_2|$ or $|Q_1|+|Q_2| - 2|Q_1 \cap Q_2|$ positions. The least of the two numbers is when the two quad intersect in a line, where the line has q + 1 points, it follows that: $d = 2(q^4 – 1) /(q – 1) – 2(q +1)$. The number of rows of G is the number of distinct quad that is, by the table 1, $M = (q^6 - 1) /(q – 1)$. The number of columns of G is the number of distinct points in the dual polar space, that is by table 1, $n = (q +1)(q^2+1)(q^3+ 1)$.

(b) One row has 1 in two positions if the two points corresponding to the two columns lie in the symplecton corresponding to the given row. Since two points are either of distance 1, 2, or 3. Points of distance 3 from each other never lie in a common symplecton. Thus two columns corresponding to two points of distance 3 have no 1 in common. Since dual polar space is a strong parapolar space, then two points of distance 2 lie exactly in one symplecton. It follows that two columns that corresponding to two points of distance 2 have exactly one 1 in common. Two collinear points lie exactly in q + 1 symplecta . Every column has number of 1's equal the number of symplecta containing the point. Since the residue of a point in the dual polar space is a projective plane then the number symplecta containing one point is $(q^3 - 1) / (q – 1)$. Thus the weight of these codewords are $w=(q^3-1) / (q–1)$. It follows that we have the following constant weight code.

For example, let q = 2 then the rows of the dual point-quad incidence matrix form a constant-weight code with parameters n = 135, M = 63, d = 24 and w = 15.

It means that this code corrects 11 errors and detects 23 errors.

The columns of the dual point-quad incidence matrix form a constant-weight code with parameters n = 63, M =135, d = 8 and w = 7.

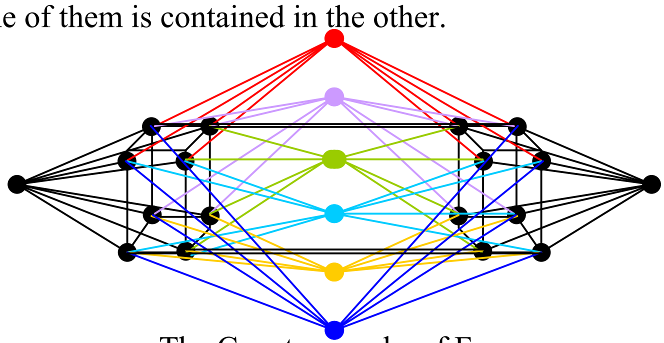
It means that this code corrects 3 errors and detects 7 errors.

Thin metasymplectic space

The thin $F_{4,1}$ geometry is the 1 - shadow space of thin F_4 geometry, it is the point - line geometry whose points are objects of type 1 and in which two points are collinear whenever they are incident to a common object of type

2. We will now present the collinearity graph of the thin $F_{4,1}$ geometry as the graph in the figure below.

The 1 - cliques, 2 - cliques, 3 - cliques and octahedral in this graph are objects of type 1, 2, 3 and 4 respectively and two objects are incident whenever one of them is contained in the other.



The Coxeter graphs of $F_{4,1}$

The thin metasymplectic space has 24 points, and 24 symplecta. symplecta has 6 points. Two symplecta either disjoint or intersect in 2 points. Every point lie in 6 symplecta. It follows that the rows or the columns of the point-symp incidence matrix forms the constant weight code with parameters:

n = 24, M = 24, d = 6, w = 6.

Point / Symp	∞	1	2	3	4	5	6	7	8	a	b	c	d	e	f	8'	7'	6'	5'	4'	3'	2'	1'	∞'
S1	1	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S2	1	0	0	1	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
S3	1	0	0	0	0	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S4	1	1	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
S5	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
S6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
S7	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	1	1	0	1
S8	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	1	1	0	0	1
S9	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	1	0	0	0	0	1
S10	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	1	1
S11	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	1	1
S12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	1
S13	0	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0	0
S14	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	1	0	0
S15	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0
S16	0	0	1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0
S17	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0
S18	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	0	1	0	1	0	0	0	0
S19	0	0	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0
S20	0	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0
S21	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0
S22	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	1	0
S23	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	1	0
S24	0	1	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	1	0

TABLE 2

Thin point-symp incidence matrix

Thick metasymplectic space.

As we have seen in the last arguments, similarly, in the thick case the number of points is the same as the number of symplecta. Two symplecta are either disjoint or intersect in a plane. Therefore we have the following result:

THEOREM. (a)*The rows of the thick point-symp incidence matrix form a constant-weight code of parameters:*

$n = M = (q^2 + q + 1)(q^3 + 1)(q^4 + 1)(q^5 + 1)$

$d = = 2(q^6 - 1)/(q - 1) - 2(q^2+q+1)$

$w = (q^6 - 1)/(q - 1)$.

(b) *The columns of the incidence matrix of the thick metasymplectic space forms a constant weight code of parameters:*

$n = M = (q^2 + q + 1)(q^3 + 1)(q^4 + 1)(q^5 + 1)$

$d = 2(q^6 - 1)/(q - 1) -2(q^2 + q + 1)$

$w = (q^6 - 1)/(q - 1)$.

Proof. (a) The numbers $n = M = (q^2 + q + 1)(q^3 + 1)(q^4 + 1)(q^5 + 1)$ is clear since the number of columns equal the number of rows. The weight w is the number of points in each symplecton, that is $w = (q^6 - 1)/(q - 1)$.

The Hamming distance of two rows is $|S_1| + |S_2| - 2|S_1 \cap S_2|$, that is either $|S_1| + |S_2|$ if the symplecta are disjoint or $|S_1| + |S_2| - 2|\text{plane}|$, the minimum of these two is $|S_1| + |S_2| - 2|\text{line}|$, so $d = 2(q^6 - 1)/(q - 1) - 2(q^2+q+1)$.

(b) $n = M = (q^2 + q + 1)(q^3 + 1)(q^4 + 1)(q^5 + 1)$ is clear since the number of columns equal the number of rows. w is the number of symplecta containing one point, so it is the number of quads in the dual polar space that is, by table 1, $w = (q^6 - 1)/(q - 1)$.

To calculate d we have to take 4 cases:

1. If p_1, p_2 are collinear points. Then the number of symplecta containing both is the the number of symplecta containing the whole line containing them. Since the residue of the geometry at a line is a projective plane then the number of symplecta containing the line is $(q^2 + q + 1)$, it follows that in this case the Hamming distance is $2(q^6 - 1)/(q - 1) - 2(q^2 + q + 1)$.
2. If p_1, p_2 are of distance two and they are a special pair then there is no common symplecton. Then the number of symplecta containing both is 0, it follows that in this case the Hamming distance is $2(q^6 - 1)/(q - 1)$.
3. If p_1, p_2 are of distance two and they are a polar pair then there is exactly one common symplecton. Then the number of symplecta containing both is 1, it follows that in this case the Hamming distance is $2(q^6 - 1)/(q - 1) - 2$.

4. If p_1, p_2 are of distance three, then there is no common symplecton. Then the number of symplecta containing both is 0, it follows that in this case the Hamming distance is $2(q^6 - 1) /(q - 1)$.

This means that the minimal of these distances is:

$$2(q^6 - 1) /(q - 1) - 2(q^2 + q + 1).$$

What is good about this family of codes is that they are easy to decode.

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