The Islamic University Journal (Series of Natural Studies and Engineering)
Vol.15, No. 1, pp 147 -155 , **2007**, ISSN 1726-6807, http// www.iugaza.edu.ps/ara/research

Nonlinear Electromagnetic TM Surface Waves in Magnetic Superlattices(LANS)Film

Dr.H.M.Mousa
Physics Department, Al.Azhar University, Gaza,
Gaza Strip, Palestinian Authority,
E.mail: h.mousa@alazhar-gaza.edu

Abstract The propagation characteristics of nonlinear TM surface waves at a lateral antiferromagnetic/nonmagnetic superlattices (LANS) film and a nonlinear dielectric cover have been investigated. LANS are linear frequency- dependent gyromagnetic media. They are described with an effective- medium theory. It is found that the frequency – wave index variation increases with the magnetic fraction f_1. We also calculate and illustrate the variation of the wave index with the power flow for various values of f_1. We found that f_1 increases the power of the nonlinear TM surface waves.

Keywords: Nonlinear Waves, Wave-guides, Dispersion relation, Magnetic Superlattices.

1. Introduction

During the last few years, nonlinear behavior of electromagnetic waves in antiferromagnetic films have attracted a significant degree of attention[1]. At the present time little seems to be known about solutions of Maxwell's equations that describe the propagation of surface or guided waves in nonlinear structures that involve linear gyromagnetic media. In addition almost all of the exact studies of TM nonlinear surface waves have been based on frequency – independent dielectric constants and attention has focused upon the infrared region of the spectrum.

Boardman and Shabat et al[2] had studied nonlinear TM surface waves along a single interface of a linear ferrite substrate and nonlinear magnetic cladding. They found that TM waves can propagate even if such propagation is in the linear, low-power medium. Wang et al[3] also studied nonlinear TM surface waves on a structure of antiferromagnetic and linear ferromagnetic media. They found that the interface could supports the nonlinear TM surface waves. Hamada et. al[4] studied nonlinear TM surface waves on a structure of nonlinear antiferromagnetic medium and linear superconductor substrate. They found that the variation of the frequency and power flow of TM waves with wave index is temperature dependent.

In this paper, we investigate the propagation characteristics of nonlinear surface waves at LANS superlattice film which are described with an

effective medium theory. Such description is valid when the wave length of the excitations are much longer than the superlattice period where $kL \leq\leq 1$, where k is the magnitude of the wave vector and $L=L_1+L_2$, is the period of the superlattice, L_1 and L_2 are the thickness of the antiferromagnetic layers and non-magnetic layers, respectively [5].
The magnetic fractions of the LANS superlattices are introduced as:

$$f_1 = \frac{L_1}{L} \quad \text{and} \quad f_2 = \frac{L_2}{L},$$

and are called the magnetic and non-magnetic fractions respectively, where $f_1 + f_2 = 1$.

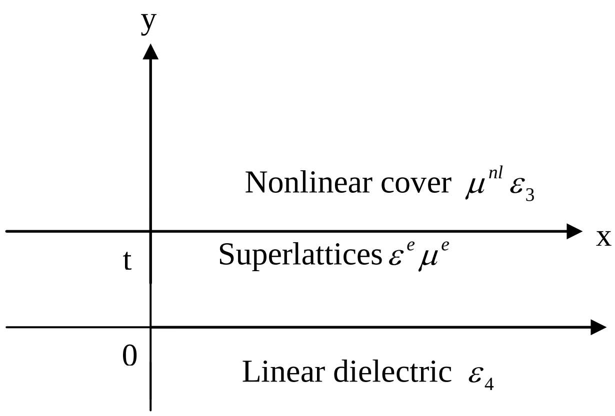

Fig..(1) TM surface waves waveguide composed of (LANS) layered structure .

2. Basic Equations

The guiding structure considered here is shown in Fig. (1). In this structure a superlattice film (LANS) of finite thickness (t) is sandwiched by a semi infinite nonlinear cladding y > t and a semi infinite linear dielectric substrate in the region y < 0. The effective dielectric tensor of (LANS) is described as[6]:

$$\varepsilon^e = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\perp \end{pmatrix}$$

Where

$$\varepsilon_\perp = \varepsilon_1 f_1 + \varepsilon_2 f_2 \quad (1a)$$

$$\varepsilon_\parallel = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 f_2 + \varepsilon_2 f_1} \quad (1b)$$

The electric and magnetic field vectors for TM waves propagating along x-axis in the xy- plane with an angular frequency ω and a wave vector k_x will take the form: (2)

$$\vec{H} = [0,\ 0,\ H_z(\omega, z)] \exp i(k_x x - \omega t)$$
$$\vec{E} = [E_x(\omega, y),\ E_y(\omega, y),\ 0] \exp i(k_x x - \omega t) \quad (3)$$

In the presence of the infra-red field associated with a TM wave propagating along the interface , the non-linear permeability of an isotropic magnetic cladding is given by (4)

$$\mu^{NL} = \mu_L + \alpha H_z^2$$

This expression arises from an expansion of the permeability about the applied static field H_o. Hence H_z is the ac magnetic field carried by the TM wave, μ_L is the linear part of the permeability and α is the non-linear coefficient.. H_z is also real because only stationary, non-radiating waves will be considered[7,8].

The wave equation in each layer is obtained from Maxwell's equations:

$$\nabla \times \vec{E} = i \omega \mu_0 \mu^{NL} \vec{H} \quad (5)$$

$$\nabla \times \vec{H} = -i \omega \varepsilon_0 \varepsilon_3 \vec{E} \quad (6)$$

where ε_3 is the relative permitivity. Substituting equations (2),(3) into (5) and equation (6) yields the following three differential equations in the three layers:

$$, \qquad y>t, \ \frac{\partial^2 H_z}{\partial y^2} - \left(k_3^2 - k_0^2 \varepsilon_3 \alpha H_z^2\right) H_z = 0 \quad (7)$$

$$\frac{\partial^2 H_z}{\partial y^2} - \left(\frac{\varepsilon_\perp}{\varepsilon_\parallel} k_x^{\ 2} + k_0^2 \varepsilon_\perp\right) H_z = 0\ , \qquad 0 \le y \le t\ , \quad (8)$$

$$\frac{\partial^2 H_z}{\partial y^2} - \left(k_x^{\ 2} - k_0^2 \varepsilon_4\right) H_z = 0\ , \qquad y< 0\ , \quad (9)$$

where $n_x = \frac{k_x}{k_0}, k_0^2 = \frac{\omega^2}{c^2} = \varepsilon_o \mu_o \omega^2, \varepsilon_o$ and μ_o are the dielectric permitivity and magnetic permeability of free space respectively .

An appropriate solution of equations has the form[2]:

1- In nonlinear cover:

$$H_z = \frac{1}{k_0}\sqrt{\frac{2}{\alpha \varepsilon_3}}\, k_3 \operatorname{sec}h\left[k_3\left(y - y_0\right)\right], \qquad (10)$$

where $k_3 = k_0\sqrt{n_x^2 - k_0^2 \varepsilon_3 \mu_l}$, (11)

and y_0 is a constant of integration that defines the position of a self focused peak in H_z .

2. In superlattice (LANS)layer

$$H_z = A_1 \sinh(\beta y) + A_2 \cosh(\beta y) \qquad (12)$$

where $\beta = k_0\sqrt{\frac{\varepsilon_\perp}{\varepsilon_\parallel} n_x{}^2 - \varepsilon_\perp}$ (13)

3.In linear dielectric substrate

$$H_z = Ce^{k_1 y} \qquad (14)$$

where $k_1 = k_0\sqrt{n_x^2 - \varepsilon_4}$. (15)

Here, A_1, A_2 are the field amplitudes in (LANS) and C is the field amplitude in the dielectric which can be determined by the boundary conditions.

By requiring the tangential components of E_x *and* H_z at the boundary y = t as:

$$\frac{k_3^2}{\varepsilon_3 k_0}\sqrt{\frac{2}{\alpha\varepsilon_3}}\sec h\left(k_3\ (t-y_0)\right)\tanh\left(k_3\ (t-y_0)\right)=\frac{\beta}{\varepsilon_\perp}\left[A_1\cosh(\beta t)+A_2\sinh(\beta t)\right] \qquad (16)$$

$$,\ \frac{k_3}{k_0}\sqrt{\frac{2}{\alpha\varepsilon_3}}\ \sec h\left(k_3\ (t-y_0)\right)=\left[A_1\sinh(\beta t)+A_2\cosh(\beta t)\right], \qquad (17)$$

Continuity of H_z *and* E_x at y = 0, yields the following equations:

$$\frac{\beta A_1}{\varepsilon_\perp}=\frac{k_1 C}{\varepsilon_4}, \qquad (18)$$

$$A_2 = C, \qquad (19)$$

Equations (18) and (19) can then yield

$$A_1=\frac{k_1\varepsilon_\perp}{\varepsilon_4\beta}A_2 \qquad (20)$$

By dividing Eq.(16) over Eq.(17) and using Eq.(20), the dispersion equation is then obtained as:

$$\tanh(\beta t)=\frac{\beta(k_3\varepsilon_4\varepsilon_\perp v-k_1\varepsilon_3\varepsilon_\perp)}{\beta^2\varepsilon_4\varepsilon_3-k_1k_3\varepsilon_\perp^2 v} \qquad (21)$$

$$where \quad v=\tanh\left[k_3(y_0-t)\right],$$

is called the magnetic nonlinearity

TM waves power flux

The total power flux(p) of the waves propagating along the x direction is:

$$p=\frac{1}{2}\int_{-\infty}^{\infty}E_z H_y\,dy \qquad (22)$$

$$= p_{NL} + p_{\sup} + p_{die}\ ,$$

where p_{NL} , $p_{\sup}$ *and* p_{co} are respectively the power fluxes in the nonlinear cover, superlattice and dielectric media and given by:

$$p_{NL} = \frac{2k_x k_3}{\omega\varepsilon_0 \alpha \varepsilon_3^2 k_0^2}\left[1-\tanh(k_3(t-y_0))\right], \quad (23a)$$

$$p_{\sup} = \frac{k_x A_2^2}{\omega\varepsilon_0 \varepsilon_{II}}\left[\begin{array}{l}\left(\frac{k_1\varepsilon_\perp}{\varepsilon_4\beta}\right)^2\left(\frac{\cosh(\beta t)\sinh(\beta t)-\beta t}{2\beta}\right) + 2\frac{k_1\varepsilon_\perp}{\varepsilon_4\beta}\left(\frac{\sinh^2(\beta t}{2\beta}\right)+ \\ \left(\frac{\cosh(\beta t)\sinh(\beta t)+\beta t}{2\beta}\right)\end{array}\right], \quad (23b)$$

$$p_{die} = \frac{k_x A_2^2}{2\omega\varepsilon_0\varepsilon_4 k_1} \quad (23c)$$

From Eq.(17) one obtains

$$A_2^2 = \frac{k_3^2}{k_0^2\varepsilon_3\alpha}\left(1-v^2\right)\left[\left(\frac{k_1\varepsilon_\perp}{\varepsilon_4\beta}\right)\sinh(\beta t)+\cosh(\beta t)\right]^{-2}, \quad (23d)$$

3. Numerical results and Discussion

To compute the dispersion curves directly, we first solve the dispersion equations numerically, this is done by fixing the parameter y_0 which is the location of the maximum in the non-linear function, given by equation (10) then roots of equation (21) are found by varying n_x which is chosed according to the following conditions:

$$\text{and } n_x >> \sqrt{\mu_L\,\varepsilon_3} \quad n_x >> \sqrt{\mu_3\,\varepsilon_{||}}$$

Numerical calculations for dispersion curves are found, examples of the dispersion curves are computed for a lateral FeF_2/ZnF_2 super lattice and non-linear material consists of a suspension of short graphite fibers in heptane and oil. We take the parameters as follows[5]:

$m_0 = 0.56\,kG$, $H_a = 200\,kG$, $H_e = 540\,kG$, $\gamma = 1.97\times10^7\ rad/\sec.G$ and $\varepsilon_1 = 5.5$ for antiferromagnetic layers, $\varepsilon_2 = 8$ for the nonmagnetic layers, $\varepsilon_3 = 2.25$, $\mu_L = 1.29$ for the non-linear medium and $\varepsilon_4 = 3$ for the substrate[4].

The propagation of the TM surface waves is reciprocal where $\omega(k_x) = \omega(-k_x)$. The frequency – wave index variation for different values of the magnetic fraction f_1 is demonstrated in Fig.(2). It shows optical instability behavior[9] i.e. for curve of label (1) where f_1 = 0.4, it shows that for the wave angular frequency $\omega = 1*10^{16}$ rad /sec there are two values of

n_x (2.8, 2.9). The optical instability is affected by the magnetic fraction f_1 where the wave velocity increases by increasing f_1.

The frequency- wave index variation for different values of the magnetic nonlinearity is shown in Fig.(3). It displays that the wave frequency is affected by the magnetic nonlinearity . The wave frequency increases by decreasing the magnetic nonlinearity

Once the propagation characteristics are determined from the dispersion equation (21), the obtained values of the refractive index can be fed to the power expression mentioned in Eq. (22).As illustrated in Fig.(4) the normalized P/P_0 has been plotted against n_x for different values of f_1 . It illustrates the dependence of the normalized power on the magnetic fraction where the increasing of the magnetic fraction causes increasing of the power. Fig (5) shows a typical field distribution H_Z as a function of the distance from the interface . If the wave index is increased, however, the maximum of the field is established at a smaller distance from the interface y = 0.

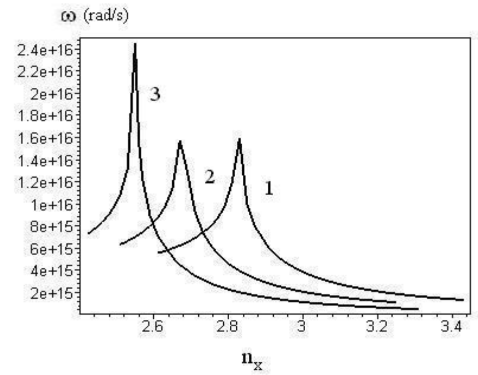

Fig..2. Dispersion curves of TM surface waves for H_0=.2kG, and v= 0.16, t =0.44×10^{-7} m (1) f_1 = 0.4, (2) f_1 = 0.6, and (3) f_1 = 0.8. The curves are labelled with values of $\alpha = 1.55\times10^{10}$ m^2 V^{-2} , μ_l =1.29 ε_3 = 2.25, ε_1 =5.5, ε_2 = 8 and ε_4= 3.

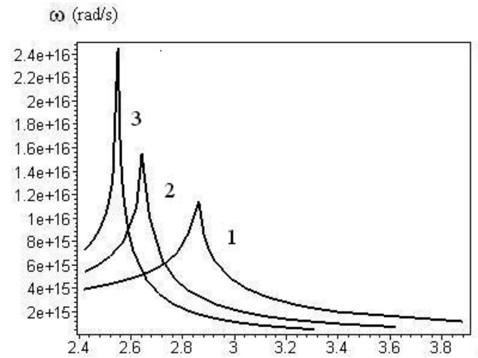

Fig.3 Dispersion curves of TM surface waves for H_0=.2kG, and f_1= 0.8, t =0.44×10^{-7} m (1) v = 0.25, (2) v = 0.2, and (3) v = 0.16. The curves are labelled with values of α = $1.55\times10^{10}\,m^2\,V^{-2}$, μ_l =1.29 ε_3 = 2.25, ε_1 =5.5, ε_2 = 8 and ε_4= 3.

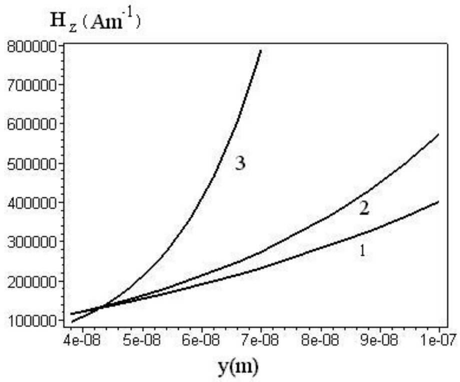

Fig. 4. Normalized TM waves power flow along the x-direction as a function of wave index (n_x) for t = 0.44×10^{-7} m, v = 0.16 (1) f_1 = 0.5, (2) f_1 =0.6, (3) f_1 = 0.7.

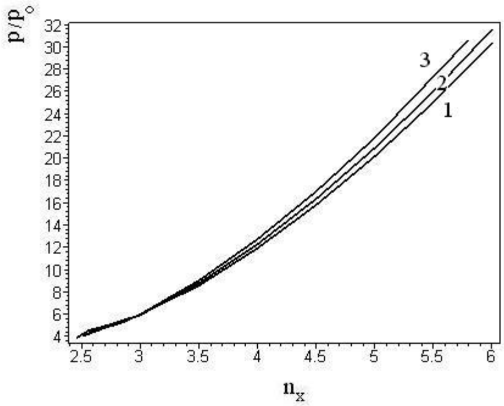

Fig. 5. Variation of TM field component across LANS film for , $f_1 = 0.8$, $t = 0.44\times10^{-7}$ m, v = 0.16 (1)n_x =2.482 $\omega = 0.914 \text{ x } 10^{16}$ rad/s (2) n_x =2.51 ω =0.1084 x 10^{17} rad/s and (3) n_x =2.55, $\omega = 0.245 \text{ x } 10^{17}$ rad/s, the other data as in Fig .2.

4. Conclusions

The TM power flow is dependent on the magnetic fraction. Magnetic fraction increases the power levels needed to observe strong nonlinear waves. By increasing the wave index, the magnetic field distribution concentrates near the interface in the nonlinear medium. We believe that the carried work will lead to future promising application in microwave-infrared technology.

5. References

1.Q, Wang and I. Awai, J. Appl. Phys. **83**, 382, (1998).

2. A. D. Boardman, M. M. Shabat, and R. F. Wallis, Phys. Rev. B,vol. **41(1)**, 717-730, (1990).

3. Q, Wang, Z. Wu, S. Li and L. Wang, J.Appl. Phys. **87**,1908, (2000).

4. M.S.Hamada, M.M.Shabat, M.M.Abd Elaal, and D.Jager, J. Superconductivity Incorporating Novel Magnetism,vol.**16(2),** 443, (2003).

5. X.Wang, and D.R Tilley, Phys. Rev. B, vol. **52**, 13-353, (1995)

6. N. S. Almeida and D. L. Mills Phys. Rev. B. **38 ,** 6698, (1988).

7. A. D. Boardman, A .A. Maradudin, G. L.Stegeman, T. wardowski, and E.M Wright, Phys. Rev. A.**35,** 1159-64, (1987).

8. A. D. Boardman, T.Twardowski,, G. L.Stegeman, and A, Shivarora, IEE. Proc. J, 134, 152- 60, (1987).

9. H.M.Mousa and M.M.Shabat, International Journal of Modern Physics,vol **19(29) ,** 4359-4369, (2005).