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ABSENCE OF FERROMAGNETISM IN ISING MODEL ON DIRECTED BARABASI-ALBERT NETWORK

M. A. Sumour, Physics Department, Al-Aqsa University,P.O.4051, Gaza, Gaza Strip, Palestinian Authority.
M. M. Shabat, Physics Department, Islamic University, P.O.108, Gaza, Gaza Strip, Palestinian Authority.
D. Stauffer, Institute for theoretical Physics, Cologne University,D-50923 Köln,Euroland.
msumoor@yahoo.com, shabat@iugaza.edu.ps, Stauffer@thp.uni-koeln.de

غياب ظاهرة المغناطيسية في نموذج أيزنق من خلال شبكية بربايسي –ألبرت المباشرة

ملخـص: في هذا البحث قمنا بتعميم نتائجنا السابقة للمغناطيسية المتقلبة للمغازل الالكترونية،وذلك بزيادة عدد المغازل لأكثر من 7 مليون مغزل وبتغيير درجة الحرارة وإجراء حسابات هذه التقلبات فـي كل زمن من خلال استخدام طريقة مونت كارلو، و التي تبدأ عندها المغناطيسية بالاضمحلال، ومن ثم التحقق من النتائج من خلال فحص قانون أرهينيس و قانون القدرة لهذه المغازل.

Abstract: With up to 7 million spins, the existence of spontaneous magnetization of Ising spins on Directed Barabasi –Albert network is investigated by Monte Carlo simulations. We confirm our earlier result that the systems magnetization for different temperatures T decays after a characteristic time (tau) t (T), which is extrapolated to diverge at zero temperature, by a modified Arrhenius law, or perhaps a power law.

Keywords: Monte Carlo simulations, Directed Barabasi–Albert network.

Introduction:

The Ising magnet is since decades a standard tool of computational physics [1]. We apply it here to scale –free networks [2], where previous simulation [3] indicated a Curie temperature increasing logarithmically with increasing system size N. In contrast to that work we use here directed [4] as opposed to undirected networks and then apply the standard Glauber kinetic Ising model [1] to the fixed network. We try to improve our previous note [5] by making the system an order of magnitude larger and smaller size, with larger time, and comparing network with two and seven neighbours.

Directed Barabasi-Albert network [4]:

Putting Ising spin onto the sites (vertices, nodes) of the network, we simulate our Ising model on directed Barabasi-Albert networks.

The Barabasi-Albert network is grown such that the probability of a new site to be connected to one of the already existing sites is proportional to the number of previous connections to this already existing site: The rich get richer.

In this way each new site selected exactly m old sites as neighbours.

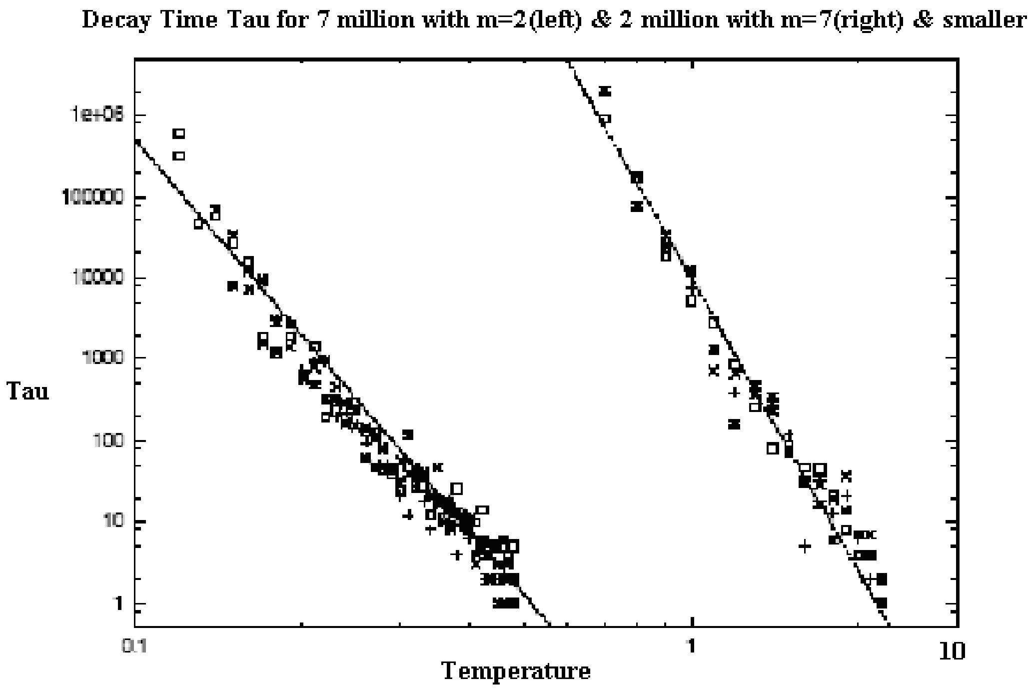

Fig. (1): Characteristic time for $M(t) = 3/4$ using 7 million spins for m=2 neighbours, and 2 million spins for m =7 neighbours (+). Ten, hundred and thousand times smaller systems are denoted by x, stars, and squares. We plot the median over nine samples in this log-log plot. The two straight lines have negative slopes 8 (left) and 12 (right).

Then each spin is influenced by the fixed number m of neighbours which it had selected when joining the network. It is not influenced by other spins which selected it as neighbour after it joined the network.

At each step, a new spin is added which builds m new connections neighbours randomly to already existing spins. The probability for an existing spin which is chosen as neighbour, is proportional to the number of its neighbours.

Ising simulations:

The Ising interaction energy is:

$$E = -J\sum_{i}\sum_{k} S_i S_k \qquad (S_i = \pm 1)$$

Where the sum over k goes only over the m neighbours which site i had selected when it joined the network. We measure the temperature in units of the usual Curie temperature of the square –lattice Ising model.

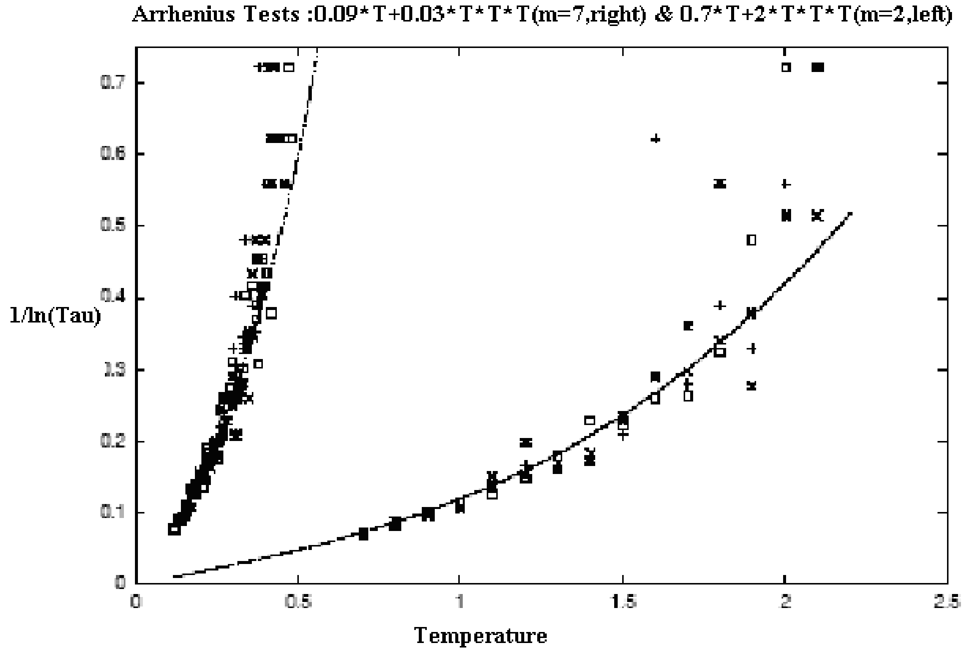

Fig. (2): some data (except for short times) plotted as 1/ln t versus T.

First we initialize a directed Barabasi-Albert network of N sites with m neighbours (all m initial spins are connected with each other and themselves), here m = 2, 7. We put an Ising spins onto every site, with all spins up, because we test here ferromagnetism. Then with the standard Glauber (heat bath) Monte Carlo algorithm spins search for thermal equilibrium at positive temperature. Time t is measured in Monte Carlo steps MCS per spins.

As shown earlier [5] the magnetization M (t) as a function of time t shows strong fluctuations and sometimes very rapid changes in a very short interval. These problems did not improve when we increase and decrease N from half a million [5] to different value now, with less statistics for the larger system, by making the system an order of magnitude larger, and smaller size, with larger time.

Now the magnetization reduces to a temperature-dependent metastable value after the first time step, stays there for a long time, and then flips or tries to flip. We follow [5] and define a characteristic time (tau) t as that time where the magnetization has decreased to 3/4 of its initial value, Fig.1. This double-logarithmic plot first suggests a power-law divergence t (T→0) → ∞. But we see slight curvature, unusually high exponents near 10, and an exponent varying with the number m= 2 or 7 of neighbours. More plausible thus is the modified Arrhenius plot of Fig. (2), which suggest $t \propto$exp (const. /T) for low temperatures, or 1/ln (t)$\propto$ T+… This constant in the exponential is about 1.4 for m=2 and about 11 for m=7.

Conclusion:

In this way we confirmed the asymptotic Arrhenius extrapolation

1/ln$t$ $\propto$ T of [5], meaning that for all finite temperatures the magnetization eventually vanishes: No ferromagnetism.

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