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Hamilton-Jacobi Formulation of The Scalar φ Coupled to Two FLavours of Fermions Through Yukawa Couplings

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Abstract

The scalar field coupled to two flavours of fermions through Yukawa couplings is treated as a constrained system using the Hamilton-Jacobi approach. The equations of motion are obtained as total differential equations in many variables. These equations of motion are in exact agreement with those equations obtained using Dirac's method.

معالجة نظام المجال القياسي φ المرتبط بعدد اثنين فريميون برباط يوكاوا بطريقة هاملتون جاكوبي

ملخص: باستخدام طريقة هاملتون جاكوبي تم معالجة نظام المجال القياسي φ المرتبط بعدد اثنين فريميون برباط يوكاوا.

تم الحصول على معادلات الحركة للنظام كمعادلات تفاضلية تامة ، وقد تطابقت هذه المعادلات مع المعادلات التي تم الحصول عليها بطريقة ديراك.

1 Introduction

The most common method for investigating the Hamiltonian treatment of constrained systems was initiated by Dirac[1]. The main feature of his method is to consider primary constraints first. All constraints are obtained using consistency conditions. Besides, he showed that the number of degrees of freedom of the dynamical system can be reduced. Hence, the equations of motion of constrained system are obtained in terms of arbitrary parameters.

The canonical method (or Güler's method) developed Hamilton-Jacobi formulation to investigate constrained systems [2-3]. The Hamilton-Jacobi treatment of constrained systems leads us to obtain the equations of motion as total differential equations in many variables. These equations are integrable if the corresponding system of partial differential equations is a Jacobi system. Since there are few physical examples were discussed by using Hamilton-Jacobi approach [4,7], it is still necessary to study more of them and compare the results that can be obtained by Dirac's method. For these reasons, the scalar field coupled to two flavours of fermions through Yukawa couplings will be studied using both Hamilton-Jacobi formulation and Dirac's method.

A review of the Hamilton-Jacobi approach can be introduced as follows:
If the rank of the Hess matrix

$$A_{ij} = \frac{\partial^2 \mathcal{L}(\varphi_i, \partial_\mu \varphi_i, x)}{\partial(\partial_\mu \varphi_i)\partial(\partial_\mu \varphi_j)}, \qquad \mu = n-r+1, ..., n, \qquad i,j = 1,2,\ldots,n, \tag{1}$$

is $(n-r), r < n$, then the standard definition of a linear momenta

$$\pi_\rho = \frac{\overrightarrow{\delta}\mathcal{L}}{\delta(\partial_\mu \varphi_\rho)}, \qquad \rho = 1,2,\ldots,n-r, \tag{2}$$

$$p_\nu = \frac{\overrightarrow{\delta}\mathcal{L}}{\delta(\partial_\mu \varphi_\nu)}, \qquad \nu = n-r+1,\ldots,n, \tag{3}$$

enables us to solve eq.(2) for $\partial_\mu \varphi_\rho$ as

$$\partial_\mu \varphi_\rho = \partial_\mu \varphi_\rho(\varphi, \partial_\mu \varphi_\nu, \pi_\gamma) \equiv \omega_\rho, \qquad \gamma = 1,2,\ldots,n-r. \tag{4}$$

Substituting eq.(4), into eq.(3), we obtain the constraints as

$$\mathcal{H}'_{\nu} = \pi_{\nu} + \mathcal{H}_{\nu}(x_{\mu}, \varphi_{\rho}, \pi_{\rho}) = 0, \tag{5}$$

where

$$\mathcal{H}_{\nu} = -\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\varphi_{\nu})}\bigg|_{(\partial_{\mu}\varphi_{\rho})\equiv\omega_{\rho}}. \tag{6}$$

The usual Hamiltonian $\mathcal{H}_0$ is defined as

$$\mathcal{H}_0 = -\mathcal{L} + \pi_{\rho}\omega_{\rho} - (\partial_{\mu}\varphi_{\nu})\mathcal{H}_{\nu}. \tag{7}$$

Like functions $\mathcal{H}_{\nu}$, the function $\mathcal{H}_0$ is not an explicit function of the velocities $\partial_{\mu}\varphi_{\rho}$. Therefore, the Hamilton-Jacobi function $S(x, \varphi)$ should satisfy the following set of Hamilton-Jacobi partial differential equations (HJPDE) simultaneously for an extremum of the function:

$$\mathcal{H}'_{\alpha}\left(t_{\beta},\ \varphi_{\alpha},\ \pi_i = \frac{\partial S}{\partial \varphi_i},\ \pi_0 = \frac{\partial S}{\partial t_0}\right) = 0, \tag{8}$$

where

$\alpha, \beta = 0, n-r+1, \ldots, n; \qquad \rho = 1, 2, \ldots, n-r$, and

$$\mathcal{H}'_{\alpha} = \pi_{\alpha} + \mathcal{H}_{\alpha}. \tag{9}$$

The canonical equations of motion are given as total differential equations in variables t_{β},

$$d\varphi_{\epsilon} = \frac{\overrightarrow{\delta}\mathcal{H}'_{\alpha}}{\delta\pi_{\epsilon}}\,dt_{\alpha}, \quad \epsilon = 0, 1, \ldots, n; \quad \alpha = 0, n-r+1, \ldots, n, \tag{10}$$

$$d\pi_{\rho} = -\frac{\overrightarrow{\delta}\mathcal{H}'_{\alpha}}{\delta\varphi_{\rho}}\,dt_{\alpha}, \qquad \rho = 1, \ldots, n-r, \tag{11}$$

$$dp_{\nu} = -\frac{\overrightarrow{\delta}\mathcal{H}'_{\alpha}}{\delta\varphi_{\nu}}\,dt_{\alpha}, \qquad \alpha = 0, n-r+1, \ldots, n. \tag{12}$$

$$dZ = \left(-\mathcal{H}_{\alpha} + p_{\rho}\frac{\delta\mathcal{H}'_{\alpha}}{\delta\pi_{\alpha}}\,dt_{\alpha}\right), \tag{13}$$

where

$$Z \equiv S(t_{\alpha}; \varphi_{\rho}), \tag{14}$$

being the action. Thus, the analysis of a constrained system is reduced to solve equations (10-12) with constraints

$$\mathcal{H}'_\alpha(t_\beta,\ \varphi_\rho,\ \pi_i) = 0, \qquad \alpha, \beta = 0, n-r+1, \ldots, n. \tag{15}$$

Since the equations (10-13) are total differential equations, integrability conditions should be checked. These equations of motion are integrable [3-5,8] if and only if the variations of $\mathcal{H}'_\alpha$ vanish identically that is

$$d\mathcal{H}'_\alpha = 0. \tag{16}$$

If conditions (10) do not vanish identically, then we consider them as new constraints. This procedure is repeated until a complete system is obtained.

This paper is arranged as follows: Dirac's method is used in sect.2 and Güler's method in sect.3. The paper closes with a conclusion in sect.4.

2 Dirac's method

We consider one loop order the self-energy for the scalar field φ with a mass m, coupled to two flavours of fermions with masses m_1 and m_2, coupled through Yukawa couplings described by the lagrangian

$$L = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{1}{6}\lambda\varphi^3 + \sum_i \overline{\psi}_{(i)}(i\gamma^\mu\partial_\mu - m_i)\psi_{(i)}$$
$$- g\varphi(\overline{\psi}_{(1)}\psi_{(2)} + \overline{\psi}_{(2)}\psi_{(1)}), \qquad \mu = 0, 1, 2, 3, \tag{17}$$

where λ is parameter and g constant, φ, $\psi_{(i)}$, and $\overline{\psi}_{(i)}$ are odd ones. We are adopting the Minkowski metric $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$.
Where Poisson bracket in Grassmann algebra, can be defined as

$$\{A, B\} = -(-1)^{n_A n_B}\{B, A\}, \tag{18}$$

where

$$n_A = \begin{cases} 0 & if \quad A \ \ even \\ 1 & if \quad A \ \ odd \end{cases}$$

The Lagrangian function (17) is singular, since the rank of the Hess matrix is one.
The generalized momenta (2,3)

$$p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = \partial^0 \varphi, \tag{19}$$

$$p_{(i)} = \frac{\partial L}{\partial \dot{\psi}_{(i)}} = i\overline{\psi}_{(i)}\gamma^0 = -H_{(i)}, \qquad i = 1, 2, \tag{20}$$

$$\overline{p}_{(i)} = \frac{\partial L}{\partial \dot{\overline{\psi}}_{(i)}} = 0 = -\overline{H}_{(i)}. \tag{21}$$

Where we must call attention to the necessity of being careful with the spinor indexes. Considering, as usual $\psi_{(i)}$ as a column vector and $\overline{\psi}_{(i)}$ as a row vector implies that $p_{(i)}$ will be a row vector while $\overline{p}_{(i)}$ will be a column vector.
Since the rank of the Hess matrix is one, one may solve (19) for $\partial^0\varphi$ as

$$\partial^0 \varphi - p_\varphi \equiv \omega. \tag{22}$$

The usual Hamiltonian H_0 is given as

$$H_0 = -L + \omega p_\varphi + \partial_0 \psi_{(i)}\, p_{(i)} \Big|_{p_{(i)} = -H_{(i)}} + \partial_0 \overline{\psi}_{(i)}\, \overline{p}_{(i)} \Big|_{\overline{p}_{(i)} = -\overline{H}_{(i)}}, \tag{23}$$

or

$$\begin{aligned} H_0 = \frac{1}{2}(p^2{}_\varphi - \partial_a \varphi \partial^a \varphi) + \frac{1}{2} m^2 \varphi^2 + \frac{1}{6}\lambda \varphi^3 - \overline{\psi}_{(i)}(i\gamma^a \partial_a - m_i)\psi_{(i)} \\ + g\varphi(\overline{\psi}_{(1)}\psi_{(2)} + \overline{\psi}_{(2)}\psi_{(1)}), \qquad a = 1, 2, 3. \end{aligned} \tag{24}$$

Eqs. (20), and (21) lead to the primary constraints

$$H'_{(i)} = p_{(i)} + H_{(i)} = p_{(i)} - i\,\overline{\psi}_{(i)}\,\gamma^0 = 0, \tag{25}$$

and

$$\overline{H}'_{(i)} = \overline{p}_{(i)} + \overline{H}_{(i)} = \overline{p}_{(i)} = 0, \tag{26}$$

respectively. These constraints lead to the total Hamiltonian

$$H_T = H_0 + \lambda_{(i)} H'_{(i)} + \overline{\lambda}_{(i)} \overline{H}'_{(i)}, \tag{27}$$

or

$$\begin{aligned} H_T = {} & \frac{1}{2}(p^2{}_{\varphi} - \partial_a \varphi \partial^a \varphi) + \frac{1}{2} m^2 \varphi^2 + \frac{1}{6} \lambda \varphi^3 - \overline{\psi}_{(i)} (i\gamma^a \partial_a - m_i) \psi_{(i)} \\ & + g\varphi(\overline{\psi}_{(1)} \psi_{(2)} + \overline{\psi}_{(2)} \psi_{(1)}) + \lambda_{(i)} (p_{(i)} - i\overline{\psi}_{(i)} \gamma^0) + \overline{\lambda}_{(i)} \overline{p}_{(i)}. \end{aligned} \tag{28}$$

According to Dirac's method, the time derivative of the primary constraints should be zero, that is

$$\dot{H}'_{(1)} = \{H'_{(1)}, H_T\} = \overline{\psi}_{(1)} (i\overleftarrow{\partial}_a \gamma^a + m_1) + g\,\varphi \overline{\psi}_{(2)} - i\overline{\lambda}_{(1)} \gamma^0 \approx 0, \tag{29}$$

$$\dot{H}'_{(2)} = \{H'_{(2)}, H_T\} = \overline{\psi}_{(2)} (i\overleftarrow{\partial}_a \gamma^a + m_2) + g\,\varphi \overline{\psi}_{(1)} - i\overline{\lambda}_{(2)} \gamma^0 \approx 0, \tag{30}$$

$$\dot{\overline{H}}'_{(1)} = \{\overline{H}'_{(1)}, H_T\} = -(i\gamma^a \partial_a - m_1) \psi_{(1)} + g\varphi\, \psi_{(2)} - i\gamma^0\, \lambda_{(1)} \approx 0, \tag{31}$$

$$\dot{\overline{H}}'_{(2)} = \{\overline{H}'_{(2)}, H_T\} = -(i\gamma^a \partial_a - m_2) \psi_{(2)} - g\varphi\, \psi_{(1)} - i\gamma^0\, \lambda_{(2)} \approx 0. \tag{32}$$

Eqs. (29-32) fix the multipliers $\overline{\lambda}_{(1)}$, $\overline{\lambda}_{(2)}$, $\lambda_{(1)}$, and $\lambda_{(2)}$ respectively as

$$i\overline{\lambda}_{(1)} \gamma^0 = \overline{\psi}_{(1)} (i\overleftarrow{\partial}_a \gamma^a + m_1) + g\,\varphi\, \overline{\psi}_{(2)}, \tag{33}$$

$$i\overline{\lambda}_{(2)} \gamma^0 = \overline{\psi}_{(2)} (i\overleftarrow{\partial}_a \gamma^a + m_2) + g\,\varphi\, \overline{\psi}_{(1)}, \tag{34}$$

$$i\gamma^0\, \lambda_{(1)} = -(i\gamma^a \partial_a - m_1) \psi_{(1)} + g\varphi\, \psi_{(2)}, \tag{35}$$

$$i\gamma^0\, \lambda_{(2)} = -(i\gamma^a \partial_a - m_2) \psi_{(2)} - g\varphi\, \psi_{(1)}. \tag{36}$$

Multiplying eqs.(33) and (34) from the right and eqs.(35) and (36) from the left by $-i\gamma^0$, we obtain

$$\overline{\lambda}_{(1)} = \overline{\psi}_{(1)} (\overleftarrow{\partial}_a \gamma^a - im_1)\, \gamma^0 - ig\,\varphi\, \overline{\psi}_{(2)} \gamma^0, \tag{37}$$

$$\overline{\lambda}_{(2)} = \overline{\psi}_{(2)}(\overleftarrow{\partial_a}\gamma^a - im_2)\gamma^0 - ig\,\varphi\,\overline{\psi}_{(1)}\gamma^0, \tag{38}$$

$$\lambda_{(1)} = -\gamma^0(\gamma^a\partial_a + im_1)\psi_{(1)} - i\,g\varphi\,\gamma^0\,\psi_{(2)}, \tag{39}$$

$$\lambda_{(2)} = -\gamma^0(\gamma^a\partial_a + im_2)\psi_{(2)} - ig\varphi\,\gamma^0\,\psi_{(1)}. \tag{40}$$

There are no secondary constraints. Taking suitable linear combinations of constraints, one has to find all numbers of second-class ones, there are

$$\Phi_i = H'_{(i)} = p_{(i)} - i\overline{\psi}_{(i)}\gamma^0, \qquad i = 1, 2, \tag{41}$$

$$\Phi_3 = \overline{H}'_{(1)} = \overline{p}_{(1)}, \tag{42}$$

and

$$\Phi_4 = \overline{H}'_{(2)} = \overline{p}_{(2)}. \tag{43}$$

The equations of motion are read as

$$\dot{\varphi} = \{\varphi, H_T\} = p_\varphi, \tag{44}$$

$$\dot{\psi}_{(i)} = \{\psi_{(i)}, H_T\} = \lambda_{(i)}, \tag{45}$$

$$\dot{\overline{\psi}}_{(i)} = \{\overline{\psi}_{(i)}, H_T\} = \overline{\lambda}_{(i)}, \tag{46}$$

$$\dot{p}_\varphi = \{p_\varphi, H_T\} = m^2\varphi + \frac{1}{2}\lambda\varphi^2 + g(\overline{\psi}_{(1)}\psi_{(2)} + \overline{\psi}_{(2)}\psi_{(1)}), \tag{47}$$

$$\dot{p}_{(1)} = \{p_{(1)}, H_T\} = \overline{\psi}_{(1)}(i\overleftarrow{\partial_a}\gamma^a + m_1) + g\varphi\overline{\psi}_{(2)}, \tag{48}$$

$$\dot{p}_{(2)} = \{p_{(2)}, H_T\} = \overline{\psi}_{(2)}(i\overleftarrow{\partial_a}\gamma^a + m_2) + g\varphi\overline{\psi}_{(1)}, \tag{49}$$

$$\dot{\overline{p}}_{(1)} = \{\overline{p}_{(1)}, H_T\} = -(i\gamma^a\partial_a - m_1)\psi_{(1)} - g\varphi\psi_{(2)} - i\gamma^0\lambda_{(1)}. \tag{50}$$

$$\dot{\bar{p}}_{(2)} = \{\bar{p}_{(2)}, H_T\} = -(i\gamma^a\partial_a - m_2)\psi_{(2)} + g\varphi\psi_{(1)} - i\gamma^0\lambda_{(2)}. \tag{51}$$

Differentiate eq.(44) with respect to time, and substituting from eq.(47), we get

$$\ddot{\varphi} - m^2\varphi - \frac{1}{2}\lambda\varphi^2 - g(\overline{\psi}_{(1)}\psi_{(2)} - \overline{\psi}_{(2)}\psi_{(1)}) = 0. \tag{52}$$

Substituting from eqs.(39) and (40) into eqs.(45), (50) and (51), we get:

$$(i\gamma^\mu\partial_\mu - m_1)\psi_{(1)} - g\varphi\,\psi_{(2)} = 0, \tag{53}$$

$$(i\gamma^\mu\partial_\mu - m_2)\psi_{(2)} - g\varphi\,\psi_{(1)} = 0, \tag{54}$$

$$\dot{\bar{p}}_{(i)} = 0, \qquad i = 1, 2. \tag{55}$$

From eqs.(37) and (38) into eq.(46), we have

$$\partial_0\overline{\psi}_{(1)}\; i\gamma^0 - \overline{\psi}_{(1)}\,(i\overleftarrow{\partial_a}\gamma^a + m_1) - g\,\varphi\,\overline{\psi}_{(2)} = 0, \tag{56}$$

$$\partial_0\overline{\psi}_{(2)}\; i\gamma^0 - \overline{\psi}_{(2)}\,(i\overleftarrow{\partial_a}\gamma^a + m_2) - g\,\varphi\,\overline{\psi}_{(1)} = 0. \tag{57}$$

In the following section the same system will be discussed by using Hamilton-Jacobi approach.

3 Hamilton-Jacobi method

The problem is going now to be tackled by using Hamilton-Jacobi. The set of (HJPDE) (8) reads as

$$\begin{aligned} H_0' = p_0 + H_0 = p_0 - \frac{1}{2}(p^2{}_\varphi - \partial_a\varphi\partial^a\varphi) + \frac{1}{2}m^2\varphi^2 + \frac{1}{6}\lambda\varphi^3 - \overline{\psi}_{(i)}(i\gamma^a\partial_a - m_i)\psi_{(i)} \\ + g\varphi(\overline{\psi}_{(1)}\psi_{(2)} + \overline{\psi}_{(2)}\psi_{(1)}), \end{aligned} \tag{58}$$

$$H_{(i)}' = p_{(i)} + H_{(i)} = p_{(i)} - i\,\overline{\psi}_{(i)}\,\gamma^0 = 0. \tag{59}$$

$$\overline{H}'_{(i)} = \overline{p}_{(i)} - \overline{H}_{(i)} = \overline{p}_{(i)} = 0. \tag{60}$$

Therefor, the total differential equations for the characteristic (10), (11) and (12) are:

$$d\varphi = p_\varphi d\tau, \tag{61}$$

$$d\psi_{(i)} = d\psi_{(i)}, \tag{62}$$

$$d\overline{\psi}_{(i)} = d\overline{\psi}_{(i)}, \tag{63}$$

$$dp_\varphi = \left[m^2\varphi + \frac{1}{2}\lambda\varphi^2 + g(\overline{\psi}_{(1)}\psi_{(2)} + \overline{\psi}_{(2)}\psi_{(1)})\right]d\tau, \tag{64}$$

$$dp_{(1)} = \left[\overline{\psi}_{(1)}(i\overleftarrow{\partial_a}\gamma^a + m_1) + g\,\varphi\,\overline{\psi}_{(2)}\right]d\tau, \tag{65}$$

$$dp_{(2)} = \left[\overline{\psi}_{(2)}(i\overleftarrow{\partial_a}\gamma^a + m_2) + g\,\varphi\,\overline{\psi}_{(1)}\right]d\tau, \tag{66}$$

$$d\overline{p}_{(1)} = \left[-(i\gamma^a\partial_a - m_1)\psi_{(1)} + g\varphi\psi_{(2)}\right]d\tau - i\gamma^0 d\psi_{(1)}, \tag{67}$$

$$d\overline{p}_{(2)} = \left[-(i\gamma^a\partial_a - m_2)\psi_{(2)} + g\varphi\psi_{(1)}\right]d\tau - i\gamma^0 d\psi_{(2)}. \tag{68}$$

The integrability conditions ($dH'_a = 0$) imply that the variation of the constraints $H'_{(i)}$ and $\overline{H}'_{(i)}$ should be identically zero, that is

$$dH'_{(i)} = dp_{(i)} - i\,d\overline{\psi}_{(i)}\gamma^0 = 0, \tag{69}$$

$$d\overline{H}'_{(i)} = d\overline{p}_{(i)} = 0. \tag{70}$$

The following equations of motion:
From eq.(61), we obtain

$$\dot{\varphi} = p_\varphi. \tag{71}$$

Substituting from eqs. (65) and (66) into eq. (69),we get

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$$i\partial_0\overline{\psi}_{(1)}\gamma^0 - \overline{\psi}_{(1)}(i\overleftarrow{\partial_a}\gamma^a + m_1) - g\varphi\overline{\psi}_{(2)} = 0, \tag{72}$$

$$i\partial_0\overline{\psi}_{(2)}\gamma^0 - \overline{\psi}_{(2)}(i\overleftarrow{\partial_a}\gamma^a + m_2) - g\varphi\overline{\psi}_{(1)} = 0. \tag{73}$$

Substituting from eqs. (67) and (68) into eq. (70), we have

$$(i\gamma^\mu\partial_\mu - m_1)\psi_{(1)} - g\,\varphi\,\psi_{(2)} = 0, \tag{74}$$

$$(i\gamma^\mu\partial_\mu - m_2)\psi_{(2)} - g\,\varphi\,\psi_{(1)} = 0. \tag{75}$$

One notes that the integrability conditions are not identically zero, they are added to the set of equations of motion.
From eqs.(64-66), we get the following equations of motion:

$$\dot{p}_\varphi = m^2\varphi + \frac{1}{2}\lambda\varphi^2 + g(\overline{\psi}_{(1)}\psi_{(2)} + \overline{\psi}_{(2)}\psi_{(1)}), \tag{76}$$

$$\dot{p}_{(1)} = \overline{\psi}_{(1)}(i\overleftarrow{\partial_a}\gamma^a + m_1) + g\,\varphi\,\overline{\psi}_{(2)}, \tag{77}$$

$$\dot{p}_{(2)} = \overline{\psi}_{(2)}(i\overleftarrow{\partial_a}\gamma^a + m_2) + g\,\varphi\,\overline{\psi}_{(1)}. \tag{78}$$

Substituting from eqs(74) and (75) into (67) and (68), we get

$$\dot{\mathbf{p}}_{(i)} = 0, \qquad i = 1, 2. \tag{79}$$

Differentiare eq.(71) with respect to time, and making use of (76), we have

$$\ddot{\varphi} - m^2\varphi - \frac{1}{2}\lambda\varphi^2 - g(\overline{\psi}_{(1)}\psi_{(2)} + \overline{\psi}_{(2)}\psi_{(1)}) = 0. \tag{80}$$

4 Conclusion

The scalar field coupled to two flavours of fermions through Yukawa couplings is discussed as constrained system [7] using both Dirac's and Hamilton-Jacobi methods. In Dirac's method the total Hamiltonian composed by adding the constraints multiplied by lagrange multipliers to the canonical Hamiltonian. In order to derive the equations of motion, one needs to redefine these unknown multipliers in an arbitrary way. However, in the Hamilton-Jacobi approach (or Güler's method)[2-8], there is no need to introduce lagrange multipliers to the canonical Hamiltonian, then the Hamilton-Jacobi is simpler and more economical.

In the Hamilton-Jacobi approach it is not necessary to distinguish between first-class and second-class constraints. The Hamilton-Jacobi approach always in exact agreement with Dirac's method. Both the consistency conditions and the integrability conditions lead to the same constraints. Also the equations of motion in both approaches are the same.

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