IUG Journal of Natural Studies
Peer-reviewed Journal of Islamic University-Gaza

ISSN 2409-4587

IUGNES
Vol. 25, No 2, 2017, pp 345-350

Accepted on (14-03-2017)

Numerical Simulation of Acoustic Waves in Metal Medium Using FDTD Method

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Acoustic wave modeling is very important for many applications such as the manufacturing of biometric sensors and detectors for flaws in metal structures. The simulation of acoustic waves is done using a finite difference time domain method (FDTD) method with absorbing conditions as perfect matching layer (PML). In this work a numerical simulation of acoustic wave's propagation in metal is presented using Matlab software program. The characteristics of simulation for the boundary condition are discussed. Results are generated and studied for 1D and 2D FDTD simulation of acoustic waves, also the velocity and pressure in x-y plane is evaluated. Our theoretical results are illustrated by various curves. This mode is important for many applications such as acoustic sensors.

Keywords:
Acoustic waves,
Metal medium,
Numerical simulation,
FDTD.

1. Introduction:

Standard finite-difference methods for the scalar wave equation have been evaluated by(Geiger and Daley (2003). These implementations handle a variable-velocity subsurface and a variety of simple boundary conditions. More recently, finite difference time-domain FDTD method considered as a numerical analysis full-wave techniques used to solve problems in electromagnetics. This method can solve complicated problems it is studied by Schneider (2016). Ostashev, Juvé, and Blanc-Benon (1997) have been Studied the derivation of a wide-angle parabolic equation for sound waves in inhomogeneous moving media. They explained several effects in outdoor sound propagation. Propagation of acoustics waves from an initial Gaussian pulse over an impedance ground and over a distance of 500 m has been studied by Dragna, Blanc-Benon, and Poisson (2010). Three impedance models classically used for applications in outdoor sound propagation are considered. Cotté, Blanc-Benon, Bogey, and Poisson (2009). Larsson and Abrahamsson (2003) have been studied a model for sound propagation which give by the time dependent acoustic wave equation. They found that an extra modeling is necessary to simulate meteorological effects like attenuation properly. Simulating different types of ground (such as woods) will also require validation against measurements. However, the wave equation has also been considered too time-consuming in real 3-D settings. The PML technique for the numerical absorption of waves, studied since greater than twenty years ago. Perfectly matched layers (PML) using for simulating the absorption of waves in open domains, which provides a very efficient alternative to the use of absorbing boundary conditions in many application Bécache, Fauqueux, and Joly (2003).

According to Etter (2013), an acoustic model is called physical or analytical when it represents the theoretical of the physical phenomena. Geiger and Daley (2003) have studied the 2D standard acoustic wave finite difference method. It requires spatial discretization of the cell size lower than λ/10 to avoid dispersion. Most of the acoustic applications have been investigated standard finite difference that is able to exhibit good overview of both stability and accuracy methods for the past two decades Zahari, Dahlan, and Madun (2015). Also, the Effects of metal buffer layer on characteristics of surface acoustic waves in metal/diamond structures discussed by Chiang, Sung, and Ro (2010), for use in the design surface acoustic wave devices in the super-high-frequency. Simulation results indicate that adding a metal buffer layer with a finite thickness significantly increases the coupling coefficient. Furthermore, Simulation of acoustic wave propagation in dispersive media with relaxation losses by using FDTD method with PML absorbing boundary condition (Yuan, Borup, Wiskin, Berggren, & Johnson, 1999). In this paper we study the modeling of acoustic wave propagation in metal media using finite difference time domain method.

2. Model and Acoustic Wave Equation:

The aim of this work is to formulate acoustic wave simulation using FDTD and to construct perfect matched layers around a 2-D square region to reduce the reflection of the acoustic wave from a point source in the middle of this region as shown in research by Hastings, Schneider, and Broschat (1996). In Figure 1 which we considered, the width of interior region is 1m, and the width of PMLs is 0.2m. The interior region is filled with metal (sound velocity c = 5130m/s, density ρ = 7800kg/m^3).

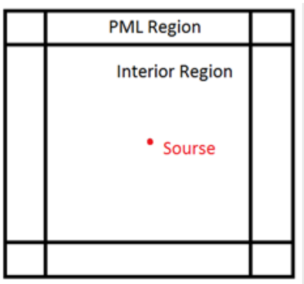

Figure 1 *Interior and PML regions*

The acoustic application of the technique considers wave propagation in metal medium using the equations for change of acoustic particle velocity v and pressure P with respect to time

$$\rho_0\rho_r \frac{\partial v}{\partial t}(x,t) = -\frac{1}{\rho}\nabla p(x,t) \qquad (1)$$

$$\kappa\frac{\partial p}{\partial t}(x,t) = \nabla\cdot v \qquad (2)$$

where ρ(x,t) is the pressure field, v(x,t) is the velocity field, ρ_0 is the mass density of water and ρ_r is the relative mass density of the medium, k= $\kappa = \frac{1}{\rho c^2}$ is the compressibility of the medium , and c is the velocity of sound and $\rho = \rho_0 \cdot \rho_r$

Equation (1) can be written as

$$\frac{dp(x,y,z,t)}{dt} = \frac{1}{\kappa(x,y,z)}\left[\frac{du_x}{dx(x,y,z,t)} + \frac{du_y}{dy(x,y,z,t)} + \frac{du_z}{dz(x,y,z,t)}\right] \qquad (3)$$

where

$$\nabla p = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right), \text{ and } \nabla\cdot v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Using first order differencing in time and space, then equation (1a) can be written

$$\frac{p^{n+\frac{1}{2}}(i,j,k) - p^{n+\frac{1}{2}}(i,j,k)}{\Delta t} =$$

$$\frac{1}{\kappa(i,j,k)}\left[\frac{u_x^n(i+\frac{1}{2},j,k) - u_x^n(i-\frac{1}{2},j,k)}{\Delta x}\right] +$$

$$\frac{1}{\kappa(i,j,k)}\left[\frac{u_y^n(i,j+\frac{1}{2},k) - u_y^n(i,j-\frac{1}{2},k)}{\Delta y}\right] + \qquad (4)$$

$$\frac{1}{\kappa(i,j,k)}\left[\frac{u_z^n(i,j+\frac{1}{2},k) - u_z^n(i,j-\frac{1}{2},k)}{\Delta z}\right]$$

Rewrite equation (4) gives a different equation as the FDTD formulation

$$p^{n+\frac{1}{2}}(i,j,k)=p^{n+\frac{1}{2}}(i,j,k)+\frac{\Delta t.\rho_0\rho_r c^2}{\Delta x}u_x^n(i+\frac{1}{2},j,k)-u_x^n(i-\frac{1}{2},j,k)$$
$$+\frac{\Delta t.\rho_0\rho_r c^2}{\Delta y}u_y^n(i,j+\frac{1}{2},k)-u_y^n(i,j-\frac{1}{2},k)+$$
$$\frac{\Delta t.\rho_0\rho_r c^2}{\Delta z}u_z^n(i,j+\frac{1}{2},k)-u_z^n(i,j-\frac{1}{2},k) \qquad (5)$$

In the same way we have the velocity equation in z direction as

$$u_z^{n+\frac{1}{2}}(i,j,k+\frac{1}{2})=u_z^{n-\frac{1}{2}}(i,j,k+\frac{1}{2})+\frac{\Delta t}{\rho_r(i,j,k+\frac{1}{2}).\rho_0.\Delta z}.[p^n(i,j,k+1)-p^n(i,j,k)] \qquad (6)$$

We choose that $c_{\max}$ in our case as a metal medium , $\Delta t \le \frac{\Delta z}{c_{\max}}$, and the velocity of present metal in our work is 5900 m/s which is nearly approximately as iron medium (5130m/s). The Equation (5,6) is solved numerically by replacing the partial derivatives by one order central finite difference expression and simulate that equations (5) and (6) using FDTD by soft ware program as Matlab.
According to suppose perfectly matched layers when a wave reaches the ends of the waveguide it is reflected back into the medium. The equations of one dimension for acoustic waves are

$$\frac{\partial v_x}{\partial t}+K\alpha v_x=-\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (7)$$

$$\frac{\partial p}{\partial t}+K\alpha p=-K\frac{\partial v_x}{\partial x} \qquad (8)$$

Where α is the attenuation coefficient. Equations (7) and (8) give the solution

$$v_x=\psi \qquad p=z\psi \qquad \psi=e^{-i\left(\frac{\omega}{c}xv\right)}e^{-Zx\alpha v} \qquad (9)$$

Let the attenuation coefficient $\alpha=0$, we get back to the original FDTD equations (1) and (2) then it will be obvious that the wave velocity is the same for medium and PML region as is the impedance

$$Z_{PML}=\frac{p}{v_x}=Z_{Medium} \qquad (10)$$

Using exponential differencing. Equation (7) becomes

$$K\alpha\frac{v_{x_{i+12}^{n+12}}-v_{x_{i+12}^{n-12}}e^{-K\alpha\delta t}}{\left(1-e^{-K\alpha\delta t}\right)}=-\frac{1}{\rho\delta x}\left\{p_{i+1}^n-p_i^n\right\} \qquad (11)$$

$$v_{x_{i+12}^{n+12}}=v_{x_{i+12}^{n-12}}e^{-K\alpha\delta t}-\frac{\left(1-e^{-K\alpha\delta t}\right)}{K\alpha}\frac{1}{\rho\delta x}\left\{p_{i+1}^n-p_i^n\right\} \qquad (12)$$

Similarly equation (8) becomes

$$\frac{K\alpha\left\{p_i^{n+1}-p_i^n e^{-K\alpha\delta t}\right\}}{\left(1-e^{-\alpha_2\delta t}\right)}=-K\left\{\frac{v_{x_{i+12}^{n+12}}-v_{x_{i-12}^{n+12}}}{\delta x}\right\} \qquad (13)$$

$$\Rightarrow p_i^{n+1}=p_i^n e^{-K\alpha\delta t}-\left(\frac{1-e^{-K\alpha\delta t}}{K\alpha}\right)K\left\{\frac{v_{x_{i+12}^{n+12}}-v_{x_{i-12}^{n+12}}}{\delta x}\right\} \qquad (14)$$

3. Results and discussion

We will discuss the use of finite difference time domain method for application as acoustic simulation. Also, we will deal with pressure waves only. Equation (13) and equation (14) have been simulated. The output is animated over time. Where in our program we choose cell size and time stepping as dx=0.1 and dt =dx/(2.*c) respectively, where c is the velocity of sound in metal medium and N is the number of time steps (100, 150, 200 and 400) which obvious in the result curves. Figure 2 shows the distribution of the pressure equation in metal medium with the wave propagation velocity c equal to 5900 m/s and for a selected frequency of 2.5 MHz using the PML. In Figure 2 the graph illustrates the simulation of acoustic waves in metal medium with acoustic velocity after 50 time steps and using ten point PML, spatial width=100 and temporal width=2. Also, we consider The source of sound locates at the center of the present supposed square. The pressure of this source is a sinusoidal function of time. Furthermore, in our software program we given some parameters which use in a loop that iterates across the waveguide to set pressures and velocities. This loop is then nested within another loop that iterates for successive time steps. We plot many graphs for different number of time steps N such as 100,150, 200 and 400 for acoustic waves in metal, In Figures 3, 4 and 5, respectively, it is shown that the reflection of PML boundary is theoretically is small , though in practice one has to give the PML boundary a width of say 10 elements and gradually introduce the absorption. We create a PML region of 10 elements and hence gradually introduce the absorption such that

$$\alpha_i = \alpha\left(\frac{x_i}{x_{PML}}\right)^2.$$

The maximum absorption for 1/10 of the PML as $e^{-K\alpha\delta t} = 0.1 \Rightarrow \alpha = \frac{1}{K\delta t}\ln(10)$, where x_i the distance is moved into PML and x_{PML} is PML width. The width of interior region is 1m, and the width of PMLs is 0.2m. The interior region is filled with metal similar to iron with (sound velocity c = 5900m/s, density ρ = 7800 kg/m3. in the FDTD methods, The property of a PML is that it is designed so that waves incident upon the PML medium do not reflect at the interface this property allows the PML to strongly absorb outgoing waves from the interior of a computational region without reflecting them back into the interior. The so PMLs don't reflect for continuous wave equation. Other simulations of the waves, the equation is discretized, so there will be numerical reflections as (Johnson, 2007). The reason the attenuation coefficient is set to increase from the inner boundary to the outer boundary is to reduce this numerical reflection. Other physical phenomena are described by coupled first-order differential equations where the temporal derivative of one field is related to the spatial derivative of another field (Kowalczyk & van Walstijn, 2011). In this work we will consider only acoustic propagation, also we choose the time step (dt) small enough to satisfy the results. According to the time steps pressure from x-y velocity have been calculated and showed in the graphs as in figures. In Figure 6 showed simulation of acoustic waves with frequency 2.5MHz and pulse is in middle and travel outward. The result of simulation with reached the perfect matching layer, it is absorbed as in Figure 8. Figure 9 illustrated that the acoustic wave were partially reflected. The Figures (6-8) represent the velocity of acoustic waves in x and Y direction and by choosing the set number of nodes in x direction and in y direction is 80 cell size.

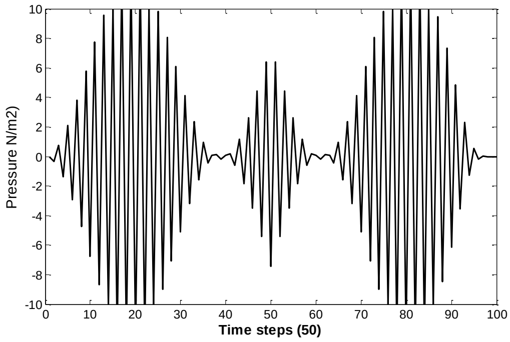

Figure 2 *Simulation of acoustic pulse in metal medium with acoustic velocity 5900m/s and density 7800kg/m³ after 50 number of time steps*

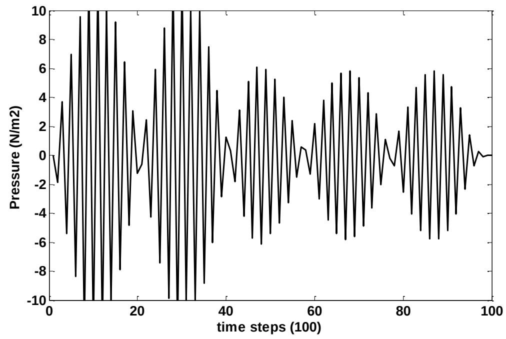

Figure 3 *Simulation of acoustic pulse in metal medium with acoustic velocity 5900m/s and density 7800kg/m³ after 100 number of time steps*

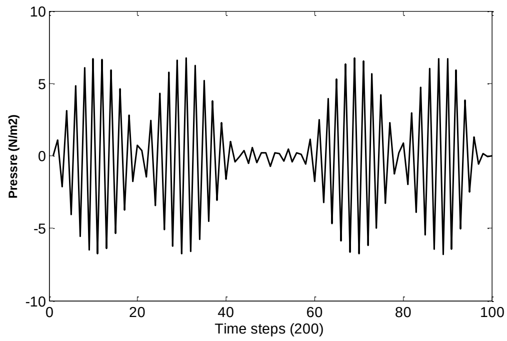

Figure 4 *Simulation of acoustic pulse in metal medium with acoustic velocity 5900m/s and density 7800kg/m³ after 200 number of time steps*

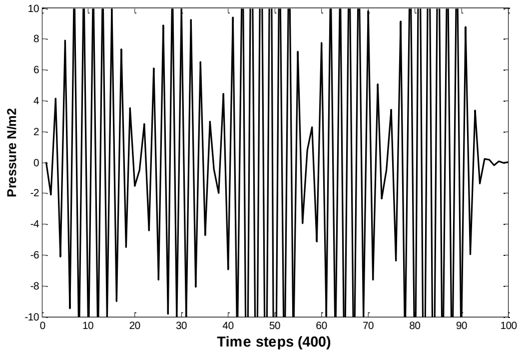

Figure 5 *Simulation of acoustic wave in metal medium with velocity 5900 m/s and density 7800kg/m³ after 400 number of time steps*

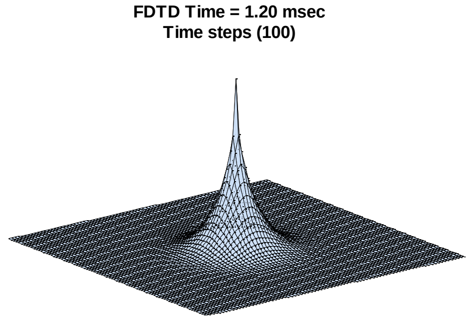

Figure 6 *Simulation of acoustic wave in metal medium with velocity 5900m/s, density 7800kg/m³, and frequency 2.5MHz, in 2D after 100 time steps*

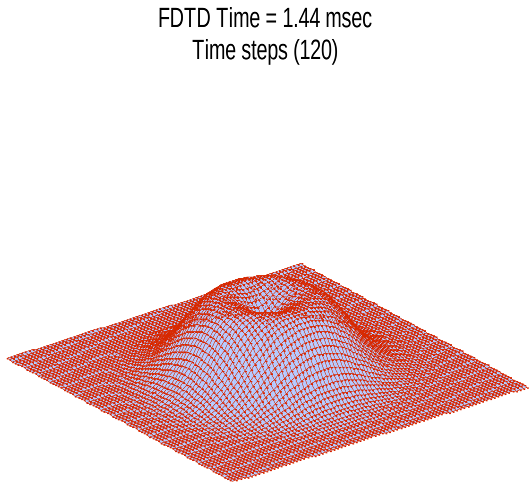

Figure 7 *Simulation of acoustic wave in metal medium with velocity 5900m/s and density 7800kg/m³, f= 2.5 MHz and time=1.44 ms after 120 time steps*

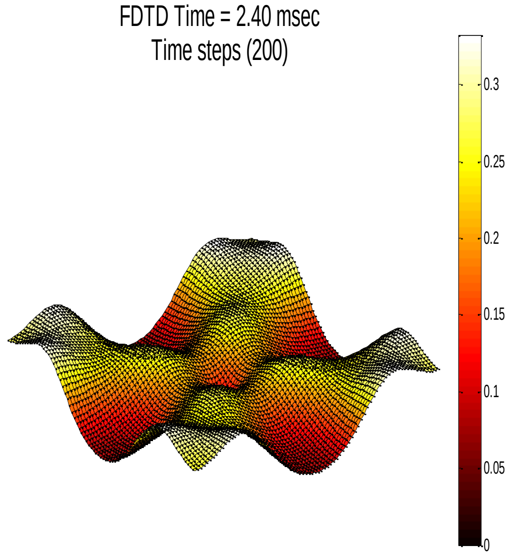

Figure 8 *Simulation of acoustic wave in metal medium with acoustic 5900m/s and density 7800kg/m³, f=2.5MHz, time=1.44 ms and after 200 time steps*

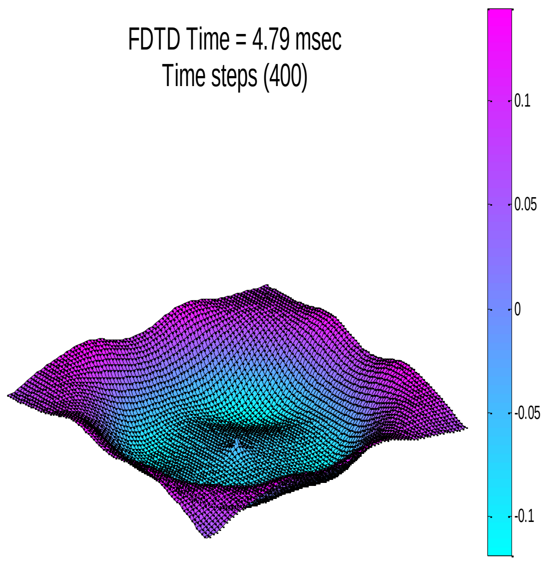

Figure 9 *Simulation of acoustic wave in metal medium with velocity 5900m/s and density 7800kg/m³ after 400 time steps, f= 2.5MHz and time=1.44 ms*

Some modifiable parameters using to plot acoustic waves in 2D that is the number of nodes in x direction equal to the number of nodes in y direction =80, the distance between nodes is dx=0.1, also we choose the time step small enough to satisfy. According to the time steps pressure from x-y velocity have been calculated and showed in the graphs as in Figures 6, 7, 8 and 9.

Conclusions:

This paper considered as a review of the modeling for acoustic propagation in metal for many time steps. Numerical FDTD simulation for acoustic wave propagation in metal medium evaluated. In addition, the characteristics of simulation for the boundary condition are discussed. Results are generated and discussed for 1D and 2D finite difference time domain method simulation of acoustic waves, also the velocity and pressure in x-y plane is evaluated. The results are illustrated by various numerical graphs with varying number of time steps. This modeling is important for many applications such as acoustic sensors. Acoustic sensors are used for the detection of pressure waves in many applications, such as communications, health monitoring and medical imaging and therapy. In addition this work may assist with a proper design, to fabricate acoustic wave sensors integrated with metal components.

References:

Bécache, E., Fauqueux, S., & Joly, P. (2003). Stability of perfectly matched layers, group velocities and anisotropic waves. *Journal of Computational Physics, 188*(2), 399-433.

Chiang, Y.-F., Sung, C.-C., & Ro, R. (2010). Effects of metal buffer layer on characteristics of surface acoustic waves in ZnO/metal/diamond structures. *Applied Physics Letters, 96*(15), 154104.

Cotté, B., Blanc-Benon, P., Bogey, C., & Poisson, F. (2009). Time-domain impedance boundary conditions for simulations of outdoor sound propagation. *AIAA Journal, 47*(10), 2391-2403.

Dragna, D., Blanc-Benon, P., & Poisson, F. (2010). *Effects of topography in time-domain simulations of outdoor sound propagation*. Paper presented at Sixteenth AIAA/CEAS Aeroacoustics Conference, Stockholm, Sweden.

Etter, P. C. (2013). *Underwater acoustic modeling and simulation*. (4th ed.). CRC Press.

Geiger, H. D., & Daley, P. F. (2003). Finite difference modelling of the full acoustic wave equation in Matlab. *Acoustic Finite Difference Modeling, CREWES Research Report, 15*, 1-9.

Hastings, F. D., Schneider, J. B., & Broschat, S. L. (1996). Application of the perfectly matched layer (PML) absorbing boundary condition to elastic wave propagation. *The Journal of the Acoustical Society of America, 100*(5), 3061-3069.

Johnson, S. (2007). Notes on Perfectly Matched Layers, online MIT course notes.

Kowalczyk, K., & van Walstijn, M. (2011). Room acoustics simulation using 3-D compact explicit FDTD schemes. *IEEE Transactions on Audio, Speech, and Language Processing, 19*(1), 34-46.

Larsson, E., & Abrahamsson, L. (2003). Helmholtz and parabolic equation solutions to a benchmark problem in ocean acoustics. *The Journal of the Acoustical Society of America, 113*(5), 2446-2454.

Ostashev, V., Juvé, D., & Blanc-Benon, P. (1997). Derivation of a wide-angle parabolic equation for sound waves in inhomogeneous moving media. *Acta Acustica united with Acustica, 83*(3), 455-460.

Schneider, J. B. (2016). *Understanding the Finite-Difference Time-Domain Method*. Retrieved http://www.eecs.wsu.edu/~schneidj/ufdtd/ufdtd.pdf

Yuan, X., Borup, D., Wiskin, J., Berggren, M., & Johnson, S. (1999). Simulation of acoustic wave propagation in dispersive media with relaxation losses by using FDTD method with PML absorbing boundary condition. *IEEE transactions on ultrasonics, ferroelectrics, and frequency control, 46*(1), 14-23.

Zahari, M., Dahlan, S. H., & Madun, A. (2015, August 10-11). *A review of acoustic FDTD simulation technique and its application to underground cavity detection*. Paper presented at International Conference on Electrical and Electronic Engineering 2015 (IC3E 2015), Melaka, Malaysia.

محاكاة عددية للموجات الصوتية في وسط معدني باستخدام طريقة المجال الزمني الفرقي المحدود

محاكاة الأمواج الصوتية مهمة جدا للعديد من التطبيقات مثل صناعة أجهزة الاستشعار وكشف البيومترية لعيوب في الإنشاءات المعدنية. يتم محاكاة الموجات الصوتية باستخدام أسلوب محدود طريقة نطاق الفارق الزمني مع شروط الامتصاص كطبقة متلاصقة مثالية تامة الامتصاص. في هذا العمل تم محاكاة عددية لانتشار الموجات الصوتية في المعادن باستخدام برنامج حاسوبي ماتلاب. خصائص المحاكاة بشروط الحدود تم مناقشتها وكذلك تم دراسة نتائج محاكاة الموجات الصوتية داخل المعدن في بعد واحد وفي بعدين باستخدام طريقة المجال الزمني الفرقي المحدود كما تم ايجاد السرعة والضغط في مستوى السيني والصادي. ويتضح لنا النتائج من خلال المنحنيات المختلفة هذا النموذج مهم للعديد من التطبيقات مثل أجهزة الاستشعار الصوتية.

كلمات مفتاحية:
الموجات الصوتية
الوسط المعدني,
المجال الفرقي,