IUG Journal of Natural Studies
Peer-reviewed Journal of Islamic University-Gaza

ISSN 2409-4587

IUGNES
Vol. 25, No 2, 2017, pp 160-166

Accepted on (14-03-2017)

Regular-Minimal and Regular-Maximal Continuous Functions

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Abstract
In this paper, we consider new types of continuous functions called regular minimal continuous and regular maximal continuous functions. We investigate their relationship with some other types of continuous functions. Also, we study further properties of the new types of continuous functions.

Keywords:
Regular open sets,
Semi-regular space,
Regular minimal continuous,
Regular maximal continuous.

1. Introduction and preliminaries:

A subset A of a topological space X is said to be a *regular open* set (Stone, 1937) if $A = Int(Cl(A))$. It is called a *regular closed* if A^c is a regular open. In Stone (1937), it was shown that the regularly open sets of a space (X,τ) is a base for a topology τ_s on X coarser than τ. The space (X,τ_s) was called *the semi-regularization* space of (X,τ). A space (X,τ) is semi-regular if and only if the regularly open sets of (X,τ) is a base for τ; that is, $\tau = \tau_s$. For a space (X,τ), the regularly open sets of (X,τ) equal the regularly open sets of (X,τ_s). Hence, the semi-regularization process generates at most one new topology. Thus $(\tau_s)_s = \tau_s$ (Stone, 1937). A space X is *connected* if X cannot be represented as the union of two or more disjoint nonempty open subsets. If X is not connected, then X is *disconnected*.

Definition 1.1 A function $f: X \to Y$ is called:

a) an *almost continuous* (Singal and Singal, 1968) if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists an open set U containing x such that $f(U) \subseteq V$.
b) an *almost perfectly continuous* (Singh, 2010) if $f^{-1}(U)$ is clopen set in X, for every regular open set U in Y.
c) an *almost strongly θ-continuous* (Noiri and Kang, 1984) if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists an open set U containing x such that $f(Cl(U)) \subseteq V$.
d) a *δ-continuous* (Noiri, 1980) if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists a regular open set U containing x such that $f(U) \subseteq V$.

A proper nonempty open (resp. closed) subset U of X is said to be a *minimal open* (resp. a *minimal closed*) set (Nakaoka and Oda, 2001) if any open (resp. closed) set which is contained in U is ϕ or U. A proper nonempty open (resp. closed) subset M of X is said to be a *maximal open* (resp. a *maximal closed*) set (Nakaoka and Oda, 2003) if any open (resp. closed) set which contains M is X or

M. The collection of all minimal open (resp. maximal open, minimal closed, maximal closed) sets is denoted by $m_iO(X)$ (resp. $M_aO(X)$, $m_iC(X)$, $M_aC(X)$).

Lemma 1.2 *(Nakaoka and Oda, 2003) Let (X,τ) be a topological space.*

a) If U is a maximal open set and W is an open set such that $U \cup W \neq X$, then $W \subseteq U$.
b) If U and V are maximal open sets such that $U \cup V \neq X$, then $U = V$.

Definition 1.3 *(Benchalli, Basavaraj, and Wali, 2011)* A topological space (X,τ) is said to be T_{min} (resp. T_{max}) space if every nonempty proper open subset of X is minimal open (resp. maximal open) set.

Remark 1.4 *(Benchalli et al., 2011) The concepts T_{min} and T_{max} spaces are identical. That is, X is T_{min} if and only if X is T_{max}.*

Definition 1.5 *(Benchalli et al., 2011)* Let X and Y be topological spaces. A map $f: X \to Y$ is called:

a) *a minimal continuous* (briefly. *a min-continuous*) if $f^{-1}(M)$ is an open set in X for every minimal open set M in Y.
b) *a maximal continuous* (briefly. *a max-continuous*) if $f^{-1}(M)$ is an open set in X for every maximal open set M in Y.

Remark 1.6 *Let Y be a T_{min} space. Then, $f: X \to Y$ is a min-continuous iff f is a max-continuous.*

Definition 1.7 *A nonempty proper regular open set A of a topological space (X,τ) is said to be:*

a) a *minimal regular open set* (Jasim and Aziz, 2014) if any regular open set contained in A is A or ϕ and a *minimal regular closed set* (Anuradha and Chacko, 2015) if any regular closed set contained in A is A or ϕ.
b) a *maximal regular open set* (Anuradha and Chacko, 2015) if any regular open set contains A is X or A and a *maximal regular closed set* (Jasim and Aziz, 2014) if any regular closed set contains A is X or A.

The collection of all minimal regular open (resp. minimal regular closed, maximal regular open, maximal regular closed) sets in a topological space (X,τ) is denoted by $m_iRO(X,\tau)$ (resp. $m_iRC(X,\tau)$, $M_aRO(X,\tau)$, $M_aRC(X,\tau)$).

Theorem 1.8 *(Mahdi and Nasser, in press) Let X be a topological space and $F \subseteq X$. Then F is a minimal regular open (resp. a minimal regular closed) set if and only if $X \setminus F$ is a maximal regular closed (resp. a maximal regular open) set.*

Lemma 1.9 *(Anuradha and Chacko, 2015) Let X be a topological space and U a minimal regular open set.*

a) If W is a regular open set such that $U \cap W \neq \phi$, then $U \subseteq W$.
b) If V is a minimal regular open set such that $U \cap V \neq \phi$, then $U = V$.

Theorem 1.10 *(Mahdi and Nasser, in press) If A is a minimal open set in a space X such that A is not dense in X, then $Int(Cl(A))$ is a minimal regular open.*

Theorem 1.11 *(Mahdi and Nasser, in press) Let X be a semi-regular space and U a nonempty regular open set, then the following three conditions are equivalent:*

a) U is a minimal regular open.
b) $U \subseteq Cl(S)$ for any nonempty subset S of U.
c) $Cl(S) = Cl(U)$ for any nonempty subset S of U.

Theorem 1.12 *(Mahdi and Nasser, in press) Let A be a nonempty subspace of X and U a regular open set in A and a regular open in X. If U is a minimal regular open in A, then U is a minimal regular open in X.*

Theorem 1.13 *(Mahdi and Nasser, in press) If A is a maximal open set in a topological space X, then exactly one of the following holds:*

1. A is a maximal regular open set.
2. A is a dense set in X.

Theorem 1.14 *(Mahdi and Nasser, in press) Let X be a semi-regular space. Then, U is a minimal regular open set if and only if U is a minimal open set; that is, $m_iO(X) = m_iRO(X)$.*

Theorem 1.15 *(Mahdi and Nasser, in press) Let (X,τ_s) be the semi-regularization space of a topological space (X,τ). Then, $m_iRO(X,\tau) = m_iRO(X,\tau_s)$ and $M_aRO(X,\tau) = M_aRO(X,\tau_s)$.*

Definition 1.16 *(Anuradha and Chacko, 2015)* A topological space (X,τ) is said to be an rT_{min} space if every proper nonempty regular open subset of X is minimal regular open.

Theorem 1.17 *(Mahdi and Nasser, in press) Let X be an rT_{min} space. Then, $m_iRO(X) = M_aRO(X)$.*

2. R-min and r-max continuous functions; definitions and characterizations:

Definition 2.1 Let X and Y be topological spaces. A map $f : X \to Y$ is called:

a) a *regular minimal continuous* (briefly, an *r-min-continuous*) if $f^{-1}(U)$ is open set in X, for every minimal regular open set U in Y.

b) a *regular maximal continuous* (briefly, *an r-max-continuous*) if $f^{-1}(U)$ is open set in X, for every maximal regular open set U in Y.

Example 2.2 *Let* $X = Y = \{1,2,3,4\}$ *with the topology* $\tau_X = \{\phi, X, \{1\}, \{4\}, \{1,4\}, \{1,2\}, \{1,2,4\}, \{1,3,4\}\}$ *and* $\tau_Y = \{\phi, Y, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,4\}, \{1,2,3\}\}$ *Then* $RO(Y, \tau_Y) = \{\phi, Y, \{1\}, \{2\}, \{4\}, \{1,4\}, \{2,4\}, \{1,2,3\}\}$. *Define* $f : X \to Y$ *by* $f(1) = 1$, $f(2) = 1$, $f(3) = 3$, $f(4) = 4$ *and* $g : X \to Y$ *by* $g(1) = 1$, $g(2) = 3$, $g(3) = 3$, $g(4) = 2$. *Then,* f *is r-min-continuous and* g *is r-max-continuous.*

Theorem 2.3 *For a function* $f : X \to Y$, *the following are equivalent:*

a) f is r-min-continuous (resp. r-max-continuous).

b) The inverse image of any maximal regular closed (resp. minimal regular closed) set is closed set.

c) For any $x \in X$ and any minimal regular open (resp. maximal regular open) set U in Y containing $f(x)$, there exists an open set W in X containing x such that $f(W) \subseteq U$.

Proof. $(a \Rightarrow b)$ The complement of minimal regular open (resp. maximal regular open) set is maximal regular closed (resp. minimal regular closed) set. Moreover, $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$.

$(b \Rightarrow c)$ If U is a minimal regular open set in Y such that $f(x) \in U$, then $Y \setminus U$ is a maximal regular closed in Y and $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is closed set in X and so $f^{-1}(U)$ is open set in X. Since $f(x) \in U$, $x \in f^{-1}(U)$. Take $W = f^{-1}(U)$, we get $x \in W = f^{-1}(U)$ and $f(W) \subseteq U$.

$(c \Rightarrow a)$ Let U be a minimal regular open set in Y. Let $x \in f^{-1}(U)$. Then there exists an open set W in X such that $x \in W$ and $f(W) \subseteq U$. Hence, $x \in W \subseteq f^{-1}(U)$ and so $f^{-1}(U)$ is open in X. □

Theorem 2.4 *Let* X, Y *be topological spaces. Consider the following two statements:*

1. Every minimal regular open set U in Y is an intersection of finitely many of maximal regular open sets in Y.
2. If $f : X \to Y$ is r-max-continuous function, then f is r-min-continuous function.

Then (1) implies (2).

Proof. Follows from the fact that $f^{-1}(\bigcap_{i=1}^{n} M_i) = \bigcap_{i=1}^{n} f^{-1}(M_i)$ and the fact that any finite intersection of open sets is open. □

Theorem 2.5 *Let* Y *be an* rT_{min} *space. Then* $f : X \to Y$ *is r-min-continuous if and only if* f *is r-max-continuous.*

Proof. Follows from Corollary 1.17. □

3. Relations between r-min, r-max and other types of continuous functions:

Firstly, there is no relation between r-min-continuous (resp. r-max-continuous) and min-continuous (resp. max-continuous) functions, as shown in the following examples:

Example 3.1 *Let* (X, τ_X) *and* (Y, τ_Y) *be as in Example 2.2. Define* $h : Y \to X$ *by* $h(1) = 2$, $h(2) = 1$, $h(3) = 1$, $h(4) = 4$ *and* $k : Y \to X$ *by* $k(1) = 4$, $k(2) = 1$, $k(3) = 2$, $k(4) = 3$. *Then,* h *is both r-min-continuous and r-max-continuous, but:*

1. h is not min-continuous, since the set $\{1\}$ is minimal open in X, but $f^{-1}(\{1\}) = \{2,3\}$ which is not open in Y.
2. h is not max-continuous, since the set $\{1,3,4\}$ is maximal open in X, but $f^{-1}(\{1,3,4\}) = \{2,3,4\}$ which is not open in Y.

Moreover, k is both min-continuous and max-continuous, but k is not r-min-continuous and not r-max-continuous since the set $\{1,2\}$ is both minimal regular open and maximal regular open in X, but $k^{-1}(\{1,2\}) = \{2,3\}$ which is not open in Y.

Theorem 3.2 *Let* Y *be a semi-regular space. Then,* $f : X \to Y$ *is r-min-continuous if and only if* f *is min-continuous.*

Proof. Follows from Theorem 1.14. □

Definition 3.3 *(Anuradha and Chacko, 2015)* Let X and Y be topological spaces. A function $f:X\to Y$ is called:

a) a *minimal regular continuous* (briefly, *a min r-continuous*) if $f^{-1}(U)$ is regular open set in X, for every minimal regular open set U in Y.

b) a *maximal regular continuous* (briefly, a *max r-continuous*) if $f^{-1}(U)$ is regular open set in X, for every maximal regular open set U in Y.

Remark 3.4 *Every min r-continuous (resp. max r-continuous) is r-min-continuous (resp. r-max-continuous), but the converse need not be true as shown in the following example:*

Example 3.5 *Let* (X,τ_X) *and* (Y,τ_Y) *be as in Example 2.2. Define* $h:X\to Y$ *by* $h(1)=1$, $h(2)=1$, $h(3)=3$, $h(4)=1$. *Then* f *is r-min-continuous, but not min r-continuous since* $\{1\}$ *is minimal regular open set in* Y, *while* $f^{-1}(\{1\})=\{1,2,4\}$ *which is not regular open in* X. *In Example 2.2,* $g:X\to Y$ *is r-max-continuous, but not max r-continuous since* $\{1,4\}$ *is maximal regular open in* Y, *while* $g^{-1}(\{1,4\})=\{1\}$ *which is not regular open in* X.

Theorem 3.6 *Let* X, Y *be topological spaces and* $f:X\to Y$.

1. If f is continuous, then f is both r-min-continuous and r-max-continuous.
2. If f is almost continuous, then f is both r-min-continuous and r-max-continuous.
3. If f is almost perfectly continuous, then f is both r-min-continuous and r-max-continuous.
4. If f is almost strongly θ continuous, then f is both r-min-continuous and r-max-continuous.
5. If f is δ-continuous, then f is both r-min-continuous and r-max-continuous.

Remark 3.7 *The converse of all parts of Theorem 3.6 need not be true as shown in the following example:*

Example 3.8 *In Example 2.2,* f *is r-min-continuous, but:*

1. f is not continuous, since $\{1,2,3\}$ is open in Y, but $f^{-1}(\{1,2,3\})=\{1,2,3\}$ which is not open in X.
2. f is not almost continuous, since $\{1,2,3\}$ is regular open set in Y, but $f^{-1}(\{1,2,3\})=\{1,2,3\}$ which is not open in X.
3. f is not almost perfectly continuous, since $\{1\}$ is regular open in Y, but $f^{-1}(\{1\})=\{1,2\}$ which is not clopen in X.
4. f is not almost strongly θ continuous, since for $x=2$ in X, $f(2)=1$, we have $\{1,4\}$ is a regular open set in Y containing $f(2)$, but there is no open set W in X contains 2 such that $f(Cl(W))\subseteq\{1,4\}$.
5. f is not δ-continuous, since for $x=2$ in X, $f(2)=1$, we have $\{1,4\}$ is a regular open set in Y containing $f(2)$, but there is no open set W in X contains 2 such that $f((W))\subseteq\{1,4\}$.

Moreover, g is r-max-continuous, but:

1. g is not almost perfectly continuous, since $\{2\}$ is regular open in Y, but $g^{-1}(\{2\})=\{4\}$ which is not clopen in X.
2. g is not almost strongly θ continuous, since for $x=4$ in X, $g(4)=2$ and $\{2,4\}$ is a regular open set in Y containing $g(4)$, but there is no open set W in X contains 4 such that $g(Cl(W))\subseteq\{2,4\}$.
3. g is not δ-continuous, since for $x=3$ in X, $g(3)=3$, we have $\{1,2,3\}$ is a regular open set in Y containing $f(3)$, but there is no regular open set W in X contains 3 such that $g((W))\subseteq\{1,2,3\}$.

Also, In Example 3.1, h is r-max-continuous, but h is not continuous since the set $\{1\}$ is open in X, but $h^{-1}(\{1\})=\{2,3\}$ which is not open in Y.

Theorem 3.9 *Let* $f:X\to Y$ *be a function such that for every* $x\in X$, *there exists a minimal regular open set* U *containing* $f(x)$. *Then* f *is r-min continuous if and only if* f *is almost continuous.*

Proof. Let $x\in X$ and G a regular open set containing $f(x)$. Then, there exists a minimal regular open set U containing $f(x)$. By Lemma 1.9, $U\subseteq G$. As f r-min continuous, there exists an open set W containing x such that $f(W)\subseteq U\subseteq G$. Therefore, f is almost continuous. The converse follows directly from Theorem 3.6 part (2). □

4. Properties of r-min and r-max continuous functions:

Theorem 4.1 *Let $f: X \to Y$ be an r-min-continuous and $x \in X$. If U is a minimal open set containing $f(x)$ such that U is not dense in Y, then there exists an open set W in X containing x such that $f(W) \subseteq Int(Cl(U))$.*

Proof. By Theorem 1.10, $Int(Cl(U))$ is a minimal regular open set in Y and $f(x) \in Int(Cl(U))$. By Theorem 2.3, there exists an open set W containing x such that $f(W) \subseteq Int(Cl(U))$. □

Theorem 4.2 *Let Y be a semi-regular space and $f: X \to Y$ a surjection function. If f is r-min-continuous, then for any minimal regular open set U in Y and any nonempty subset S of U, there is a nonempty open set W in X such that $W \subseteq f^{-1}(Cl(S))$.*

Proof. Let U be a minimal regular open set in Y and S a nonempty subset of U. Then $W = f^{-1}(U)$ is a nonempty open set. By Theorem 1.11, $U \subseteq Cl(S)$. So, $f(W) \subseteq U \subseteq Cl(S)$. Therefore, $W \subseteq f^{-1}(Cl(S))$. □

Theorem 4.3 *Let $f: X \to Y$ be an r-min-continuous (resp. an r-max-continuous). Then for any subset A of X, $f|A: A \to Y$ is an r-min-continuous (resp. an r-max-continuous).*

Proof. Direct from the fact that $(f|A)^{-1}(U) = f^{-1}(U) \cap A$. □

Theorem 4.4 *Let $\{A_\alpha : \alpha \in \Delta\}$ be an open cover of X. Then, $f: X \to Y$ is r-min-continuous (resp. r-max-continuous) if and only if $f|A_\alpha : A_\alpha \to Y$ is r-min-continuous (resp. r-max-continuous), $\forall \alpha \in \Delta$.*

Proof. If $f: X \to Y$ is r-min-continuous (resp. r-max-continuous), then by Theorem 4.3, $f|A_\alpha : A_\alpha \to Y$ is r-min-continuous (resp. r-max-continuous), $\forall \alpha \in \Delta$. Conversely, assume that $\forall \alpha \in \Delta$, $f|A_\alpha : A_\alpha \to Y$ is r-min-continuous. Let U be a minimal regular open set in Y. Then, $f^{-1}(U) = \bigcup_{\alpha \in \Delta}((f|A_\alpha)^{-1}(U))$. But $(f|A_\alpha)^{-1}(U)$ is open set in A_α for every $\alpha \in \Delta$. Since A_α is open in X, $(f|A_\alpha)^{-1}(U)$ is open set in X and so, $\bigcup_{\alpha \in \Delta}((f|A_\alpha)^{-1}(U)) = f^{-1}(U)$ is open in X. Therefore, $f: X \to Y$ is r-min-continuous. Similarly, if $f: X \to Y$ is r-max-continuous. □

Corollary 4.5 *Let $X = A \cup B$ where A and B are both open (or both closed) sets in X. Then, a function $f: X \to Y$ is r-min-continuous (resp. r-max-continuous) iff both $f|A$ and $f|B$ are r-min-continuous (resp. r-max-continuous).* □

Theorem 4.6 *Let B be a subspace of a space Y and $f: X \to B$. If $RO(B) \subseteq RO(Y)$ and $f: X \to Y$ is r-min-continuous, then $f: X \to B$ is r-min-continuous.*

Proof. If U is a minimal regular open set in B, then by Theorem 1.12, U is a minimal regular open set in Y and so the result follows. □

Remark 4.7 *The composition of even two r-min-continuous (resp. r-max-continuous) functions need not be r-min-continuous (resp. r-max-continuous).*

Example 4.8 *Let (X, τ_X), (Y, τ_Y) be as in Example 2.2 and $Z = \{a, b, c\}$ with $\tau_Z = \{\phi, Z, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Define $f: X \to Y$ as in Example 2.2 and $h: Y \to Z$ by $h(1) = b$, $h(2) = c$, $h(3) = b$, $h(4) = a$. Then f and h are r-min-continuous. But $g \circ f: X \to Z$ is not r-min-continuous since the set $\{b, c\}$ is minimal regular open in Z, while $(g \circ f)^{-1}(\{b, c\}) = f^{-1}(g^{-1}(\{b, c\}))$ $= f^{-1}(\{1,2,3\}) = \{1,2,3\}$ which is not open in X.*

Example 4.9 *Let $X = Y = Z = \{1,2,3,4\}$ with $\tau_X = \{\phi, X, \{1\}, \{2\}, \{1,2\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}$, $\tau_Y = \{\phi, Y, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{2,4\}\}$ and $\tau_Z = \{\phi, Z, \{1\}, \{4\}, \{1,4\}, \{1,2\}, \{1,2,4\}, \{1,3,4\}\}$. Then $RO(Y, \tau_Y) = \{\phi, Y, \{1\}, \{3\}, \{1,3\}, \{1,2,4\}, \{2,3,4\}, \{2,4\}\}$ and $RO(Z, \tau_Z) = \{\phi, Z, \{4\}, \{1,2\}\}$. Define $f: X \to Y$ by $f(1) = 1$, $f(2) = 3$, $f(3) = 2$, $f(4) = 4$ and $g: Y \to Z$ by $g(1) = 1$, $g(2) = 2$, $g(3) = 3$, $g(4) = 3$. Then, both f and g are r-max-continuous, but $g \circ f: X \to Z$ is not r-max-continuous since $\{1,2\}$ is maximal regular open set in Z, but $(g \circ f)^{-1}(\{1,2\}) = f^{-1}(g^{-1}(\{1,2\})) = f^{-1}(\{1,2\}) = \{1,3\}$ which is not open set in X.*

The proofs of the following three theorems are direct and trivial, so they are omitted.

Theorem 4.10 *Let $f: X \to Y$ be a continuous function and $g: Y \to Z$ an r-min-continuous (resp. an r-max-continuous). Then $g \circ f: X \to Z$ is r-min-continuous (resp. r-max-continuous).*

Theorem 4.11 *Let* $f: X \to Y$ *be an almost continuous and* $g: Y \to Z$ *a min r-continuous (resp. a max r-continuous). Then* $g \circ f: X \to Z$ *is r-min-continuous (resp. r-max-continuous).*

Theorem 4.12 *Let* Y *be a* T_{min} *space. If* $f: X \to Y$ *is a min-continuous and* $g: Y \to Z$ *is an r-min-continuous (resp. an r-max-continuous), then* $g \circ f: X \to Z$ *is r-min-continuous (resp. r-max-continuous).*

Theorem 4.13 *Let* Z *be a semi-regular space. If* $f: X \to Y$ *is a continuous and* $g: Y \to Z$ *is a min-continuous, then* $g \circ f: X \to Z$ *is r-min-continuous.*

Proof. Follows from Theorem 3.2 and Theorem 4.10. □

Theorem 4.14 *Let* $f: X \to Y$ *be a surjction open mapping. If* $g: Y \to Z$ *is a function such that* $g \circ f: X \to Z$ *is r-min-continuous (resp. r-max-continuous), then* $g: Y \to Z$ *is r-min-continuous (resp. r-max-continuous).*

Proof. Let U be a minimal regular open set in Z, then $(g \circ f)^{-1}(U)$ is open in X and so $f((g \circ f)^{-1}(U))$ $= f(f^{-1}(g^{-1}(U)))$ $= g^{-1}(U)$ is open in Y. □

Theorem 4.15 *Let* (X, τ_s) *and* (Y, σ_s) *be the semi-regularization spaces of topological spaces* (X, τ) *and* (Y, σ)*, respectively. Then,*

a) $f:(X,\tau) \to (Y,\sigma)$ is r-min-continuous (resp. r-max-continuous) if and only if $f:(X,\tau) \to (Y,\sigma_s)$ is r-min-continuous (resp. r-max-continuous).

b) If $f:(X,\tau_s) \to (Y,\sigma)$ is r-min-continuous (resp. r-max-continuous), then $f:(X,\tau) \to (Y,\sigma)$ is r-min-continuous (resp. r-max-continuous).

Proof. (a) Follows directly from Theorem 1.15.

(b) Follows from the fact that $\tau_s \subseteq \tau$. □

Theorem 4.16 *Let* A *and* B *be two minimal regular open sets in* Y *such that* $A \cup B = Y$. *If* $f: X \to Y$ *is r-min-continuous, then* X *is disconnected.*

Proof. Since $A \neq B$, by Lemma 1.9, $A \cap B = \phi$. So, $f^{-1}(A)$ and $f^{-1}(B)$ are two open sets in X. Since $A \cap B = \phi$ and $A \cup B = Y$, then $f^{-1}(A) \cap f^{-1}(B) = \phi$ and $f^{-1}(A) \cup f^{-1}(B) = X$. □

Theorem 4.17 *Let* M_1 *and* M_2 *be two disjoint maximal open sets in a space* Y *such that they are not dense. If* $f: X \to Y$ *is r-max-continuous, then* X *is disconnected.*

Proof. By Theorem 1.13, M_1 and M_2 are two maximal regular open sets. As $f: X \to Y$ is r-max-continuous, $f^{-1}(M_1)$ and $f^{-1}(M_2)$ are two open sets in X. By Lemma 1.2, $M_1 \cup M_2 = Y$. Hence, $f^{-1}(M_1) \cup f^{-1}(M_2) = X$ and $f^{-1}(M_1) \cap f^{-1}(M_2) = \phi$. □

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الدوال المتصلة المنتظمة العظمى والمنتظمة الصغرى

في هذا البحث، قمنا بتقديم أنواع جديدة من الدوال المتصلة تسمى الدوال المتصلة المنتظمة العظمى والدوال المتصلة المنتظمة الصغرى. قمنا بالبحث في علاقاتهما بالأنواع المختلفة من الدوال المتصلة. إضافة لذلك فقد قمنا بدراسة خصائص إضافية لأنواع جديدة أخرى من الدوال المتصلة.

كلمات مفتاحية:
المجموعات المفتوحة المنتظمة،
الفضاء شبه المنتظم،
الدوال المتصلة المنتظمة العظمى،
الدوال المتصلة المنتظمة الصغرى.