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HAMILTONIAN QUANTIZATION OF HIGHER-ORDER EFFECTIVE LAGRANGIAN WITH MASSIVE VECTOR FIELDS

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Abstract

Hamilton−Jacobi approach for higher−order Lagrangian density is discussed. The quantization of Yang−Mills theory with interaction term is obtained by this procedure after we converted it to the first order one.

1 Introduction

The study of singular systems has a wide range in physics, since the development of the Hamiltonian formulation by Dirac [1, 2]. The higher−order singular Lagrangian systems were studied in many works as in refs.[3, 4, 5, 6]

Most of physical systems can be described by Lagrangian that depends at most on the first derivatives of the dynamical variables. There is a continuing interest in the so called generalized dynamics, that is, the study of physical systems described by Lagrangians containing derivatives of higher order than the first. The study of higher order Lagrangian was developed by Ostrogradisky[7, 8].
In fact, this work is an application of previous paper [9], where we have studied the Hamilton−Jacobi approach for higher−order singular Lagrangian density.

The general form of Euler−Lagrange equation of motion with n order is given by [8]

$$\frac{\partial \mathcal{L}}{\partial A^{\epsilon}_{d}} - \partial^{\lambda}\left(\frac{\partial \mathcal{L}}{\partial(\partial^{\lambda} A^{\epsilon}_{d})}\right) + \cdots + (-1)^{n}\partial^{\lambda_1}\ldots\partial^{\lambda_n}\left(\frac{\partial \mathcal{L}}{\partial(\partial^{\lambda_1}\ldots\partial^{\lambda_n} A^{\epsilon}_{d})}\right) = 0. \quad (1)$$

Now we want to find the general form of equation of motion of the effective Lagrangian density for Yang−Mills field. The effective Lagrangian density is expressed as [10]

$$\mathcal{L}_{eff} = \mathcal{L}_{o} + \varepsilon\mathcal{L}_{I} = -\frac{1}{4}F^{\mu\nu}_{a}F^{a}_{\mu\nu} + \frac{1}{2}M^{2}A^{\mu}_{a}A^{a}_{\mu} + \varepsilon\mathcal{L}_{\mathcal{I}}(A^{\mu}_{a}, \partial^{\nu}A^{\mu}_{a}, \ldots, \partial^{\nu_1}\cdots\partial^{\nu_n}A^{\mu}_{a}), \quad (2)$$

where $\mathcal{L}_o$ represents a massive Yang−Mills theory and the effective interaction term $\mathcal{L}_I$ contains the deviation from the Yang−Mills interactions which involve derivatives up to the order N and which are proportional to ε with $\varepsilon \ll 1$. The generalized field strength tensor $F^{\mu\nu}_{a}$ is defined by

$$F^{\mu\nu}_{a} = \partial^{\mu}A^{\nu}_{a} - \partial^{\nu}A^{\mu}_{a} + gf_{abc}A^{\mu}_{b}A^{\nu}_{c}, \quad (3)$$

where g is a constant.

The Euler-Lagrange equation of motion of the effective Lagrangian(2) is obtained as

$$0 = D^{\lambda}F^{d}_{\epsilon\lambda} + M^{2}A^{d}_{\epsilon} +$$

$$\varepsilon\left[\frac{\partial\mathcal{L}_I}{\partial A_d^\epsilon}-\partial^\lambda\left(\frac{\partial\mathcal{L}_I}{\partial(\partial^\lambda A_d^\epsilon)}\right)+\cdots+(-1)^n\partial^{\lambda_1}\ldots\partial^{\lambda_n}\left(\frac{\partial\mathcal{L}_I}{\partial(\partial^{\lambda_1}\ldots\partial^{\lambda_n}A_d^\epsilon)}\right)\right], \quad (4)$$

where $F_{\epsilon\lambda}^d=\partial_\epsilon A_\lambda^d-\partial_\lambda A_\epsilon^d+gf^{def}A_\epsilon^e A_\lambda^f$, and M is the mass.

The main aim of this paper is to solve the higher−order effective interactions of massive vector fields, using the Hamilton−Jacobi method.

2 Hamilton−Jacobi Method

The Hamilton−Jacobi approach for singular systems was developed by Güler[11, 12]. The higher−order effective Lagrangian (2) can be written as

$$\mathcal{L}_{eff}=-\frac{1}{4}(\partial^\mu A_a^\nu-\partial^\nu A_a^\mu+gf_{abc}A_b^\mu A_c^\nu)g_{\mu\lambda}g_{\nu\epsilon}g^{ad}g^{be}g^{cf}(\partial^\lambda A_d^\epsilon-\partial^\epsilon A_d^\lambda$$

$$+gf_{def}A_e^\lambda A_f^\epsilon)+\frac{1}{2}M^2A_a^\mu g_{\lambda\mu}g^{ad}A_d^\lambda+\varepsilon\mathcal{L}_I(A_a^\mu,\partial^\nu A_a^\mu,\ldots,\partial^{\nu_1}\cdots\partial^{\nu_n}A_a^\mu), \quad (5)$$

where $g^{\alpha\beta}=diag(+1,-1,-1,-1)$ is the metric tensor.
Now, we would like to convert the effective Lagrangian (5) to first order Lagrangian density, let us introduce the fields

$$B_a^{\mu,\nu_1}=\partial^\nu A_a^\mu, \qquad C_a^{\mu,\nu_1}=(B_a^{\mu,\nu_1}-\partial^\nu A_a^\mu) \quad (6)$$

$$B_a^{\mu,\nu_1,\nu_2}=\partial^{\nu_1}\partial^{\nu_2}A_a^\mu=\partial^{\nu_2}B_a^{\mu,\nu_1}, \qquad C_a^{\mu,\nu_1,\nu_2}=(B_a^{\mu,\nu_1,\nu_2}-\partial^{\nu_2}B_a^{\mu,\nu_1}) \quad (7)$$

$$\vdots$$

$$B_a^{\mu,\nu_1,\ldots,\nu_{n-1}}=\partial^{\nu_{n-1}}B_a^{\mu,\nu_1,\ldots,\nu_{n-2}},$$

$$C_a^{\mu,\nu_1,\ldots,\nu_{n-1}}=(B_a^{\mu,\nu_1,\ldots,\nu_{n-1}}-\partial^{\nu_{n-1}}B_a^{\mu,\nu_1,\ldots,\nu_{n-2}}) \quad (8)$$

$$B_a^{\mu,\nu_1,\ldots,\nu_n}=\partial^{\nu_n}B_a^{\mu,\nu_1,\ldots,\nu_{n-1}}, \;\; C_a^{\mu,\nu_1,\ldots,\nu_n}=(B_a^{\mu,\nu_1,\ldots,\nu_n}-\partial^{\nu_n}B_a^{\mu,\nu_1,\ldots,\nu_{n-1}}) \quad (9)$$

Therefore, the reduced form of the effective Lagrangian density can be written as

$$\mathcal{L}_{red}=-\frac{1}{4}(B^{\nu_1,\mu_1}-B^{\mu_1,\nu_1}+gf_{abc}A_b^\mu A_c^\nu)g_{\lambda\mu}g_{\epsilon\nu}g^{ad}g^{be}g^{cf}(B_d^{\epsilon,\lambda}-B_d^{\lambda,\epsilon}+$$

$$gf_{def}A_e^\lambda A_f^\epsilon)+\frac{1}{2}M^2A_a^\mu g_{\lambda\mu}g^{ad}A_d^\lambda+\varepsilon[\mathcal{L}_{oI}(A_a^\mu,B_a^{\mu,\nu_1},\ldots,B_a^{\mu,\nu_1,\ldots,\nu_{n-1}},$$

$$\partial^{\nu_n} B_a^{\mu,\nu_1,\ldots,\nu_{n-1}}) + C_a^{\mu,\nu_1}(B_a^{\mu,\nu_1} - \partial^{\nu} A_a^{\mu}) + C_a^{\mu,\nu_1,\nu_2}(B_a^{\mu,\nu_1,\nu_2} - \partial^{\nu_2} B_a^{\mu,\nu_1})$$
$$+ \cdots + C_a^{\mu,\nu_1,\ldots,\nu_n}(B_a^{\mu,\nu_1,\ldots,\nu_n} - \partial^{\nu_n} B_a^{\mu,\nu_1,\ldots,\nu_{n-1}})]. \tag{10}$$

The corresponding canonical momenta are

$$\pi_a^{\mu,\nu_1} = \frac{\partial \mathcal{L}_{red}}{\partial(\partial^{\nu} A_a^{\mu})} = -\varepsilon C_a^{\mu,\nu_1}, \tag{11}$$

$$\pi_a^{\mu,\nu_1,\nu_2} = \frac{\partial \mathcal{L}_{red}}{\partial(\partial^{\nu_2} B_a^{\mu,\nu_1})} = -\varepsilon C_a^{\mu,\nu_1,\nu_2}, \tag{12}$$

$$\pi_a^{\mu,\nu_1,\nu_2,\nu_3} = \frac{\partial \mathcal{L}_{red}}{\partial(\partial^{\nu_3} B_a^{\mu,\nu_1,\nu_2})} = -\varepsilon C_a^{\mu,\nu_1,\nu_2,\nu_3}, \tag{13}$$

$$\vdots$$

$$\pi_a^{\mu,\nu_1,\ldots,\nu_n} = \frac{\partial \mathcal{L}_{red}}{\partial(\partial^{\nu_n} B_a^{\mu,\nu_1,\ldots,\nu_{n-1}})} = -\varepsilon C_a^{\mu,\nu_1,\ldots,\nu_n}, \tag{14}$$

$$\pi_a^{\mu,\nu_1,\ldots,\nu_{n+1}} = \frac{\partial \mathcal{L}_{red}}{\partial(\partial^{\nu_n} C_a^{\mu,\nu_1,\ldots,\nu_n})} = 0. \tag{15}$$

The corresponding canonical Hamiltonian density is expressed as

$$\mathcal{H}^{\alpha} = \frac{1}{4}(B_a^{\nu_1,\mu_1} - B_a^{\mu_1,\nu_1} + g f_{abc} A_b^{\mu} A_c^{\nu}) g_{\lambda\mu} g_{\epsilon\nu} g^{ad} g^{be} g^{cf} (B_d^{\epsilon,\lambda} - B_d^{\lambda,\epsilon} + g f_{def} A_e^{\lambda} A_f^{\epsilon})$$
$$-\frac{1}{2} M^2 A_a^{\mu} g_{\lambda\mu} g^{ad} A_d^{\lambda} - \varepsilon \mathcal{L}_{oI}(A_a^{\mu}, B_a^{\mu,\nu_1}, \ldots, B_a^{\mu,\nu_1,\ldots,\nu_n}, \partial^{\nu_n} B_a^{\mu,\nu_1,\ldots,\nu_n}) +$$
$$B_a^{\mu,\nu_1} \pi_a^{\mu,\nu_1} + B_a^{\mu,\nu_1,\nu_2} \pi_a^{\mu,\nu_1,\nu_2} + \cdots + B_a^{\mu,\nu_1,\ldots,\nu_n} \pi_a^{\mu,\nu_1,\ldots,\nu_n} \tag{16}$$

The set of Hamilton Jacobi Partial Differential Equations (HJPDE) is

$$\mathcal{H}'^{\alpha} = \mathcal{H}^{\alpha} + \pi^{\alpha} \approx 0 \tag{17}$$

$$\mathcal{H}_e'^{\lambda,\epsilon_1} = \pi_e^{\lambda,\epsilon_1} + \varepsilon C_e^{\lambda,\epsilon_1} \approx 0 \tag{18}$$

$$\mathcal{H}_e'^{\lambda,\epsilon_1,\epsilon_2} = \pi_e^{\lambda,\epsilon_1,\epsilon_2} + \varepsilon C_e^{\lambda,\epsilon_1,\epsilon_2} \approx 0 \tag{19}$$

$$\vdots$$

$$\mathcal{H}_e'^{\lambda,\epsilon_1,\ldots,\epsilon_n} = \pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_n} + \varepsilon C_e^{\lambda,\epsilon_1,\ldots,\epsilon_n} \approx 0 \tag{20}$$

$$\mathcal{H}'^{\lambda,\epsilon_1,\dots,\epsilon_{n+1}}_{e} = \pi^{\lambda,\epsilon_1,\dots,\epsilon_{n+1}}_{e} \approx 0 \tag{21}$$

The equations of motion are obtained as total differential equations as follows,

$$dA^{\epsilon_1}_{d} = \frac{\partial(\pi^\alpha + \mathcal{H}^\alpha)}{\partial\pi^{\epsilon,\lambda_1}_{d}}dx^\alpha + \frac{\partial(\pi^{\lambda,\lambda_1}_{e} + \varepsilon C^{\lambda,\epsilon_1}_{e})}{\partial\pi^{\epsilon,\lambda_1}_{d}}dA^{\lambda_1}_{e} + \frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2}_{e} + \varepsilon C^{\lambda,\epsilon_1,\epsilon_2}_{e})}{\partial\pi^{\epsilon,\lambda_1}_{d}}dB^{\lambda,\epsilon_1}_{e}$$

$$+\frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2,\lambda_3}_{e} + \varepsilon C'^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}_{e})}{\partial\pi^{\epsilon,\lambda_1}_{d}}dB^{\lambda,\epsilon_1,\epsilon_2}_{e} + \cdots +$$

$$\frac{\partial(\pi^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e} + \varepsilon C'^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e})}{\partial\pi^{\epsilon,\lambda_1}_{d}}dB^{\lambda,\epsilon_1,\dots,\epsilon_{n-1}}_{e} + \frac{\partial\pi^{\lambda,\epsilon_1,\dots,\epsilon_{n+1}}_{e}}{\partial\pi^{\epsilon,\lambda_1}_{d}}\, dC^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e}, \tag{22}$$

$$dB^{\epsilon,\lambda_1}_{d} = \frac{\partial(\pi^\alpha + \mathcal{H}^\alpha)}{\partial\pi^{\epsilon,\lambda_1,\lambda_2}_{d}}dx^\alpha + \frac{\partial(\pi^{\lambda,\epsilon_1}_{e} + \varepsilon C^{\lambda,\epsilon_1}_{e})}{\partial\pi^{\epsilon,\lambda_1,\lambda_2}_{d}}dA^{\lambda_1}_{e} + \frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2}_{e} + \varepsilon C^{\lambda,\epsilon_1,\epsilon_2}_{e})}{\partial\pi^{\epsilon,\lambda_1,\lambda_2}_{d}}dB^{\lambda,\epsilon_1}_{e}$$

$$+\frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}_{e} + \varepsilon C'^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}_{e})}{\partial\pi^{\epsilon,\lambda_1,\lambda_2}_{d}}dB^{\lambda,\epsilon_1,\epsilon_2}_{e} + \cdots +$$

$$\frac{\partial(\pi^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e} + \varepsilon C'^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e})}{\partial\pi^{\epsilon,\lambda_1,\lambda_2}_{d}}dB^{\lambda,\epsilon_1,\dots,\epsilon_{n-1}}_{e} + \frac{\partial\pi^{\lambda,\epsilon_1,\dots,\epsilon_{n+1}}_{e}}{\partial\pi^{\epsilon,\lambda_1,\lambda_2}_{d}}\, dC^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e}, \tag{23}$$

$$\vdots$$

$$dB^{\epsilon,\lambda_1,\dots,\lambda_{n-1}}_{d} = \frac{\partial(\pi^\alpha + \mathcal{H}^\alpha)}{\partial\pi^{\epsilon,\lambda_1,\dots,\lambda_n}_{d}}dx^\alpha + \frac{\partial(\pi^{\lambda,\epsilon_1}_{e} + \varepsilon C^{\lambda,\epsilon_1}_{e})}{\partial\pi^{\epsilon,\lambda_1,\dots,\lambda_n}_{d}}dA^{\lambda_1}_{e} +$$

$$\frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2}_{e} + \varepsilon C^{\lambda,\epsilon_1,\epsilon_2}_{e})}{\partial\pi^{\epsilon,\lambda_1,\dots,\lambda_n}_{d}}dB^{\lambda,\epsilon_1}_{e} + \frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}_{e} + \varepsilon C^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}_{e})}{\partial\pi^{\epsilon,\lambda_1,\dots,\lambda_n}_{d}}dB^{\lambda,\epsilon_1,\epsilon_2}_{e} + \cdots +$$

$$\frac{\partial(\pi^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e} + \varepsilon C'^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e})}{\partial\pi^{\epsilon,\lambda_1,\dots,\lambda_n}_{d}}dB^{\lambda,\epsilon_1,\dots,\epsilon_{n-1}}_{e} + \frac{\partial\pi^{\lambda,\epsilon_1,\dots,\epsilon_{n+1}}_{e}}{\partial\pi^{\epsilon,\lambda_1,\dots,\lambda_n}_{d}}\, dC^{\lambda,\epsilon_1,\dots,\epsilon_n}_{e}, \tag{24}$$

And

$$d\pi^{\epsilon,\lambda_1}_{d} = -\frac{\partial(\pi^\alpha + \mathcal{H}^\alpha)}{\partial A^{\epsilon_1}_{d}}dx^\alpha - \frac{\partial(\pi^{\lambda,\epsilon_1}_{e} + \varepsilon C^{\lambda,\epsilon_1}_{e})}{\partial A^{\epsilon_1}_{d}}dA^{\lambda_1}_{e} -$$

$$\frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2}_{e} + \varepsilon C^{\lambda,\epsilon_1,\epsilon_2}_{e})}{\partial A^{\epsilon_1}_{d}}dB^{\lambda,\epsilon_1}_{e} - \frac{\partial(\pi^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}_{e} + \varepsilon C^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}_{e})}{\partial A^{\epsilon_1}_{d}}dB^{\lambda,\epsilon_1,\epsilon_2}_{e} - \cdots -$$

$$\frac{\partial(\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}+\varepsilon C_e^{\lambda,\epsilon_1,\ldots,\epsilon_n})}{\partial A_d^{\epsilon_1}}dB_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n-1}}+\frac{\partial\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n+1}}}{\partial A_d^{\epsilon_1}}\,dC_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}, \tag{25}$$

$$d\pi_d^{\epsilon,\lambda_1,\lambda_2}=-\frac{\partial(\pi^\alpha+\mathcal{H}^\alpha)}{\partial B_d^{\epsilon,\lambda_1}}dx^\alpha-\frac{\partial(\pi_e^{\lambda,\epsilon_1}+\varepsilon C_e^{\lambda,\epsilon_1})}{\partial B_d^{\epsilon,\lambda_1}}dA_e^{\lambda_1}-$$
$$\frac{\partial(\pi_e^{\lambda,\epsilon_1,\epsilon_2}+\varepsilon C_e^{\lambda,\epsilon_1,\epsilon_2})}{\partial B_d^{\epsilon,\lambda_1}}dB_e^{\lambda,\epsilon_1}-\frac{\partial(\pi_e^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}+\varepsilon C_e^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3})}{\partial B_d^{\epsilon,\lambda_1}}dB_e^{\lambda,\epsilon_1,\epsilon_2}-\cdots-$$
$$\frac{\partial(\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}+\varepsilon C_e^{\lambda,\epsilon_1,\ldots,\epsilon_n})}{\partial B_d^{\epsilon,\lambda_1}}dB_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n-1}}+\frac{\partial\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n+1}}}{\partial B_d^{\epsilon,\lambda_1}}\,dC_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}, \tag{26}$$

$$d\pi_d^{\epsilon,\lambda_1,\ldots,\lambda_n}=-\frac{\partial(\pi^\alpha+\mathcal{H}^\alpha)}{\partial B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}}}dx^\alpha-\frac{\partial(\pi_e^{\lambda,\epsilon_1}+\varepsilon C_e^{\lambda,\epsilon_1})}{\partial B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}}}dA_e^{\lambda_1}-$$
$$\frac{\partial(\pi_e^{\lambda,\epsilon_1,\epsilon_2}+\varepsilon C_e^{\lambda,\epsilon_1,\epsilon_2})}{\partial B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}}}dB_e^{\lambda,\epsilon_1}-\frac{\partial(\pi_e^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}+\varepsilon C_e^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3})}{\partial B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}}}dB_e^{\lambda,\epsilon_1,\epsilon_2}$$
$$-\cdots-\frac{\partial(\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}+\varepsilon C_e^{\lambda,\epsilon_1,\ldots,\epsilon_n})}{\partial B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}}}dB_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n-1}}+\frac{\partial\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n+1}}}{\partial B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}}}\,dC_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}, \tag{27}$$

$$d\pi_d^{\epsilon,\lambda_1,\ldots,\lambda_{n+1}}=-\frac{\partial(\pi^\alpha+\mathcal{H}^\alpha)}{\partial(\partial^{\lambda_n}B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}})}dx^\alpha-\frac{\partial(\pi_e^{\lambda,\epsilon_1}+\varepsilon C_e^{\lambda,\epsilon_1})}{\partial(\partial^{\lambda_n}B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}})}dA_e^{\lambda_1}-$$
$$\frac{\partial(\pi_e^{\lambda,\epsilon_1,\epsilon_2}+\varepsilon C_e^{\lambda,\epsilon_1,\epsilon_2})}{\partial(\partial^{\lambda_n}B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}})}dB_e^{\lambda,\epsilon_1}-\frac{\partial(\pi_e^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3}+\varepsilon C_e^{\lambda,\epsilon_1,\epsilon_2,\epsilon_3})}{\partial(\partial^{\lambda_n}B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}})}dB_e^{\lambda,\epsilon_1,\epsilon_2}-\cdots-$$
$$\frac{\partial(\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}+\varepsilon C_e^{\lambda,\epsilon_1,\ldots,\epsilon_n})}{\partial(\partial^{\lambda_n}B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}})}dB_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}+\frac{\partial\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n+1}}}{\partial(\partial^{\lambda_n}B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}})}\,dC_e^{\lambda,\epsilon_1,\ldots,\epsilon_n}. \tag{28}$$

The equations (22-28) are reduced to

$$dA_d^\epsilon=B_d^{\epsilon,\lambda_1}dx^{\lambda_1}, \tag{29}$$

$$dB_d^{\epsilon,\lambda_1}=B_d^{\epsilon,\lambda_1,\lambda_2}dx^{\lambda_2}, \tag{30}$$

$$dB_d^{\epsilon,\lambda_1,\lambda_2}=B_d^{\epsilon,\lambda_1,\lambda_2,\lambda_3}dx^{\lambda_3}, \tag{31}$$

$$dB_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}} = B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1},\lambda_n} dx^{\lambda_n}, \tag{32}$$

$$d\pi_d^{\epsilon,\lambda_1} = \left(M^2 A^d_{\epsilon_1} + \varepsilon \frac{\partial \mathcal{L}_{oI}}{\partial A_d^{\epsilon_1}} \right) dx^{\lambda_1}, \tag{33}$$

$$d\pi_d^{\epsilon,\lambda_1,\lambda_2} = \left(-F^d_{\epsilon\lambda} + \varepsilon \frac{\partial \mathcal{L}_{oI}}{\partial B_d^{\epsilon,\lambda_1}} - \pi_d^{\epsilon,\lambda_1} \right) dx^{\lambda_2}, \tag{34}$$

$$\vdots$$

$$d\pi_d^{\epsilon,\lambda_1,\ldots,\lambda_n} = \left[\varepsilon \frac{\partial \mathcal{L}_{oI}}{\partial B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}}} - \pi_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}} \right] dx^{\lambda_n}, \tag{35}$$

$$d\pi_d^{\epsilon,\lambda_1,\ldots,\lambda_{n+1}} = \varepsilon \frac{\partial \mathcal{L}_{oI}}{\partial (\partial^{\lambda_n} B_d^{\epsilon,\lambda_1,\ldots,\lambda_{n-1}})} dx^{\lambda_{n+1}}. \tag{36}$$

The equations (29-36) are integrable if and only if the following integrability conditions are satisfied.

$$d\ \mathcal{H}'^{\alpha} = d\ \mathcal{H}^{\alpha} + d\pi^{\alpha} \approx 0, \tag{37}$$

$$d\ \mathcal{H}_e'^{\lambda,\epsilon} = d\pi_e^{\lambda,\epsilon} + \varepsilon d\ C_e^{\lambda,\epsilon} \approx 0, \tag{38}$$

$$d\ \mathcal{H}_e'^{\lambda,\epsilon_1,\epsilon_2} = d\pi_e^{\lambda,\epsilon_1,\epsilon_2} + \varepsilon d\ C_e^{\lambda,\epsilon_1,\epsilon_2} \approx 0, \tag{39}$$

$$\vdots$$

$$d\ \mathcal{H}_e'^{\lambda,\epsilon_1,\ldots,\epsilon_n} = d\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_n} + \varepsilon d\ C_e^{\lambda,\epsilon_1,\ldots,\lambda_n} \approx 0, \tag{40}$$

$$d\ \mathcal{H}_e'^{\lambda,\epsilon_1,\ldots,\epsilon_{n+1}} = d\pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n+1}} \approx 0. \tag{41}$$

Equation (37) vanishes identically, but equations (38-41) are satisfied under the following conditions:

$$dC_e^{\lambda,\epsilon_1} = -\frac{1}{\varepsilon} \left[M^2 A^e_{\lambda_1} + \varepsilon \frac{\partial \mathcal{L}_{oI}}{\partial A_e^{\lambda_1}} \right] dx^{\epsilon_1}, \tag{42}$$

$$dC_e^{\lambda,\epsilon_1,\epsilon_2} = -\frac{1}{\varepsilon}\left[F_{\lambda\epsilon}^{e} + \varepsilon\frac{\partial \mathcal{L}_{oI}}{\partial B_e^{\lambda,\epsilon_1}} - \pi_e^{\lambda,\epsilon_1}\right] dx^{\epsilon_2}, \tag{43}$$

$$\vdots$$

$$dC_e^{\lambda,\epsilon_1,\ldots,\epsilon_n} = -\frac{1}{\varepsilon}\left[\varepsilon\frac{\partial \mathcal{L}_{oI}}{\partial B_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n-1}}} - \pi_e^{\lambda,\epsilon_1,\ldots,\epsilon_{n-1}}\right] dx^{\epsilon_n}, \tag{44}$$

$$d\pi_d^{\lambda,\epsilon_1,\ldots,\epsilon_{n+1}} = 0. \tag{45}$$

Solving equations (29-36) simultaneously, one gets

$$D^{\lambda_1}F_{\epsilon\lambda}^{d} + M^2 A_\epsilon^d + \varepsilon\left[\frac{\partial \mathcal{L}_{oI}}{\partial A_d^\epsilon} - \frac{d}{dx^{\lambda_1}}\left(\frac{\partial \mathcal{L}_{oI}}{\partial B_d^{\epsilon_1}}\right) + \frac{d}{dx^{\lambda_1}}\frac{d}{dx^{\lambda_2}}\left(\frac{\partial \mathcal{L}_{oI}}{\partial B_d^{\epsilon_1,\epsilon_2}}\right) + \cdots +\right]$$

$$\varepsilon\left[(-1)^n\frac{d}{dx^{\lambda_1}}\cdots\frac{d}{dx^{\lambda_n}}\left(\frac{\partial \mathcal{L}_{oI}}{\partial B_d^{\epsilon_1,\ldots,\epsilon_n}}\right)\right] = 0, \tag{46}$$

Equation (46) can be rewritten as

$$D^{\lambda}F_{\epsilon\lambda}^{d} + M^2 A_\epsilon^d + \varepsilon\left[\frac{\partial \mathcal{L}_I}{\partial A_d^\epsilon} - \partial^\lambda\left(\frac{\partial \mathcal{L}_I}{\partial(\partial^\lambda A_d^\epsilon)}\right) + \cdots +\right]$$

$$\varepsilon\left[(-1)^n\partial^{\lambda_1}\ldots\partial^{\lambda_n}\left(\frac{\partial \mathcal{L}_I}{\partial(\partial^{\lambda_1}\ldots\partial^{\lambda_n}A_d^\epsilon)}\right)\right] = 0. \tag{47}$$

which is the same as Eq(4).

3 Conclusion

In this paper, we investigate the higher−order effective Lagrangians of Yang−Mills fields. We dealt with it as frst−order effective Lagrangian, by introducing an auxiliary fields (as new constraints). The equations of motion are obtained as total differential equations in many variables, which satisfied the integrability conditions under certain conditions on the auxiliary fields. Simultaneous solutions of the equations of motions with constraints lead us to the same Euler−Lagrange equation of motion as obtained in ref. [10]

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